BETWEEN MATHEMATICS AND TALMUD – THE CONSTRUCTION OF A HYBRID DISCOURSE IN AN ULTRA-ORTHODOX CLASSROOM

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This paper examines the case of adult ultra-orthodox Jews studying algebra for the first time, in a pre-college course. First, the social context and Talmudic background of the students is presented. Then, we analyse how cultural resources from both the Talmudic studies, the main practice of ultra-orthodox culture, and the mathematics classroom culture, were used by students to construct a new hybrid discourse. We conclude by discussing how our analysis demonstrates some productive possibilities and potential problems for the students' mathematical learning.

INTRODUCTION

Culture has been of increasing interest to mathematics education researchers (ex. Sfard & Prusak, 2005). A particular focus has been placed on the difficulties of minority groups to participate in mathematical classroom discourse (ex. Cobb & Hodge, 2002). Less attention has been given to instances where the cultural background of students may also *enhance* productive participation in learning. Building on the linguistic anthropological notion of “hybrid discourse” (Lefstein & Snell, 2010) we analyse the discourse patterns in a classroom of ultra-orthodox adults who are being introduced to the concept of proof and the activity of mathematical proving (Stylianides, 2007). In this setting, students come from a background of highly argumentative learning (as practiced in Talmudic studies) together with almost no disciplinary knowledge in mathematics other than basic arithmetic. Our goal in this paper is to examine the intersection between mathematical classroom discourse and the cultural discourse students bring into class.

THEORETICAL BACKGROUND

In our analysis, we will adopt a *communicational* lens (Sfard, 2008), which views learning mathematics as participation in a certain type of discourse characterized by a particular set of *words*, *routines* and *narratives*. In particular, we pay attention to the *interactional routines* followed by participants (Empson, 2003; Heyd-Metzuyanim, 2013). Specifically, we look at who *initiates* an interaction, where does the *authority* for making mathematical claims lie (Veel, 1999), and in what ways *key words* are being used (Sfard, 2008).

On top of Sfard's framework, we wish to use the notion of *hybrid discourse* put forward by Lefstein and Snell (2010). Lefstein and Snell propose that discourses are dynamic processes rather than static entities. Therefor there are gaps between a discourse prototype and its realization in practice. These gaps - which stem from the individual's agency and creativity, and from the complexity of social situations - are filled by borrowing from other cultural resources, thereby constructing a hybrid discourse. In *communicational* terms, we will expect this hybridity to take place by a drift of *words, routines, narratives,* and even goals and purposes, from one discourse to the other. Yet before moving on to showing how such hybridity may take place between Talmudic and mathematical discourses, we must provide some context and background on the ultra-orthodox Talmudic culture.

The ultra-orthodox males in Israel ideally devote most of their time to Talmudic studies. In the elementary years, they receive some basic education in “secular” domains such as mathematics, English, and geography during a short period of the day. This secular education ends at the age of 12-13 after which males go into “Yeshivas”. There, only sacred texts are studied, mostly the books of the Talmud, which are often descriptions of the mythological scholars' disputes concerning older texts.

Some researchers, such as Schwarz (in press) and Segal (2011), have already drawn attention to the parallels between Talmudic learning practices and instructional practices that have been highly valued in reform efforts to turn classrooms into “discussion based” learning environments (Herbel-Eisenmann & Cirillo, 2009; Lampert, 1990). However, Schwarz and Segal also point to some attributes which are significantly different than practices prevalent in traditional pre academic mathematical classrooms. In what follows, we briefly outline these differences.

Learning purposes: Unlike pre academic mathematics, which is often studied as a preparation for more advanced mathematics courses or for reasons external to the discipline (such as admission to academic tracks), in Talmudic studies the act of learning is held up as a goal in itself .The ultra-orthodox religious values encourage engagement with the sacred texts as an activity worth of its own. In Blum-Kulka’s words:

> “The religious obligation to study the (Talmudic) law is not goal-oriented, but concerns itself merely with process. … The ideal of *Torah li-shmah (*Torah study as an end unto itself.) underscores the perception that time spent on disagreement is of the same religious value as that expended on reaching an agreement“ (Blum-Kulka 2002, p. 1576).

The structure of interactional routines: The basic and very common Talmud structure of interactional routines is that of two learners ('Havruta') who engage with a text without the constant guidance of an instructor. In contrast, the structure of the pre academic mathematics class is that of a teacher-led instruction, followed by some independent class work.

Routines for endorsement of narratives: Talmudic and mathematical discourses differ in the ways they determine a statement as “true”. In mathematics, two opposing proposition cannot be simultaneously declared as “true”. The “truth” of a statement is based on its coherence (or agreement) with all mathematical narratives that have been endorsed up to that point. Talmudic justification, on the other hand, involves reasoning between several, often equally plausible alternatives. As a consequence, one's Talmudic interpretation must be supported by evidence ("Re'aia"), but it doesn't necessarily refute other interpretations, as mathematical counter-examples do.

Authority structure: As mentioned above, Talmudic studies are often performed in pairs. While the teacher (Rabbi) is often present and has a voice in the discussion, he is not regarded as the ultimate arbitrator. In mastering the "Cultural preference for disagreement" (Blum-Kulka, ibid), students are given the authority to both disagree and put forward unique and creative arguments. In contrast, in mathematics classrooms (especially those described as “teacher-centred” or “traditional”), the authority to state what is true or false mainly lies with the teacher. Students in such classrooms are accustomed to rely on the teacher's authority and develop intricate methods of interpreting his stance, even when it is not stated explicitly. Though the “teacher as ultimate arbitrator” phenomenon has been fought against within reform attempts (ex. Lampert, 1990), it can be seen as an attribute of the pre-academic mathematical classroom discourse. In mathematics, there **is** often only one correct answer, and the teacher, being more knowledgeable than the students, mostly has access to it before they do.

In light of the above similarities and differences, we would like to closely analyse the hybrid discourse observed in our research, and to ask what discursive possibilities did it open or close for students’ participation and for the development of their mathematical discourse?

THE STUDY

The current study follows a class of pre-academic mathematics for adult ultra-orthodox males, at the ages of 18 to 30, preparing them for bachelor studies in business school. The course took place 6 hours a week (two days) during 13 months, from January 2013 to February 2014, and was taught by Nadav (the first author). At the period when the study was conducted, Nadav held a B.Sc. in mathematics & Computer-Science with only minimal formal training as a teacher. The course started from the most basic algebraic signs and methods, and ended with an exam equivalent to 3 points of 'Bagrut' exam (the basic level of the Israeli mathematics matriculation exam).

Seven students attended the lesson described below, out of ten who were enrolled in the course. During the recordings, a stationary video camera pointed at the teacher to allow a view of the board. No other cameras were in use, mainly for political reasons. The topic of ultra-orthodox integration in the modern Israeli society has been controversial in the past years. Therefore, we were careful not to steer objections to the documentation, neither by the students nor by the college management. The lesson was transcribed in Hebrew and later translated by the authors into English.

AN EPISODE OF ARGUMENTATION ABOUT MATHEMATICAL PROOF

The particular episode examined here took place in September 2013 after 8 months of instruction. It was chosen for close analysis because of the rich discussion that took place in it, and because it had seemed to contain evidence for the construction of a hybrid discourse from both ultra-orthodox and mathematics classroom cultures.

The session concerned the proving of the quadratic formula. Through that, the teacher planned to discuss what constitutes a proof in mathematics and what does not. First, he wrote on the board an example of a quadratic equation ($x^2 - 3x + 2=0$) and, with the aid of the students, established that by either factoring or using the quadratic formula, one can find that its roots are 1 and 2, and to verify it using substitution. Then the following proceeded (all names are pseudonyms):

1.	T	... In this example the quadratic formula has given two solutions that are indeed true. You found it using substitution (Joshua: yes). Does it mean it is really true? Does it mean it always finds the solutions?
2.	Joshua	Not yet. Not yet.
3.	Abraham	If I see ten twenty of those.. then.. then what?
4.	T	Then?
5.	Abraham	It's sure to be true
6.	Joshua	If you see that it's infinite. If you see that it's some infinite process that makes it always substitute the…

The above excerpt shows that some students, and in particular Abraham, started out holding a naïve, or inductive idea of proof. The more a formula is empirically checked to provide true solutions (by substituting the symbols for real numbers), the surer one is that it is "true". In contrast, Joshua caught on to the teacher's questioning of the inductive claim and declared that such substitution would not suffice [2]. Instead, one should look for an "infinite process" that would always make it true [6]. Capitalizing on another student's comment that "there's always an exception", the teacher moved to explain that in mathematics, an example is not a proof, whereas a counter-example is. Therefore, even showing hundreds of examples where a formula is correct wouldn't prove it. At this point, Abraham responded: "so you need to understand its (the formula's) logic" showing that he was moving away from the inductive reasoning.

Having established that examples cannot be considered as proof, the teacher offered a procedure for proving the quadratic formula beyond doubt. He did not, however, present it as an established proof but rather as a hypothetical procedure that *might* be considered a proof if the students accepted it.

44.	T	What of the things that I have here (points to the general quadratic equation $ax^2+bx+c=0$ and the quadratic formula written on the board) can I substitute (and) where? *(Silence)* Just like I took x_{12}… I had one and two… I substituted it here (points to $x^2 - 3x + 2=0$)

45.	Avraham	That does not prove it, we said that
46.	T	It doesn't prove it. If I take these x one and two, for instance? (*points at the quadratic formula*) (*Silence. Teacher writes the following quadratic equation on the board, while saying it out loud)* $a(\frac{-b\pm\sqrt{b^2-4ac}}{2a})^2+b(\frac{-b\pm\sqrt{b^2-4ac}}{2a})+c=0$
47.	Avraham	(*As the teacher is writing*) It's better 'cause it shows you like that for any number… It proves 'cause it shows me that any number that I'll put here..
48.	T	Good

Here occurred the main transformation in Abraham's claims about what "proof" was. Instead of claiming that "10 or 20" correct trials would prove a formula, he now sees that what the teacher was suggesting would "show it for any number" [47]. Notably, this important transformation in Abraham's claims occurred through participating in a teacher-led IRE interaction, and the idea of such a proof was completely initiated by the teacher (the students at this stage of their studies had no way of coming up with such a proof on their own). However, what unfolded next was a change in the classroom's structure of interactional routines. Concurrently with the interaction between the teacher and other students who were seeking clarifications (mostly Yehuda, who was sitting at the front desk), an episode of argumentation between Joshua and Abraham (who were sitting in the back) enabled the latter to take ownership and practice *authority* over the teacher's idea. The next excerpt concentrates on the dialogue between Joshua and Abraham:

70.	T	Instead of substituting (with) the number 2 I'll substitute the expression with letters…. (Joshua: OK), All of it , and I'll see that at the end, all of this (points to the quadratic formula) becomes zero, so that will tell me that actually this always becomes zero. Do you agree that this is a way that really shows me this formula is always correct?
71.	Joshua	As long as there's no counter example, yeah
72.	*At this point Yehuda starts asking questions and conversing with the teacher.*	
73.	Abraham	There can't be a counter example. Any number that you put will work.
74.	Joshua	Why not? And if you put an incorrect number?
75.	Abraham	But it can't be. (Joshua: Why not?) 'Cause these letters told you that any number that you substitute for them, you'll get zero.
76.	Joshua	That's if it works. And what if it won't work?
77.	Abraham	But it can't be that it wouldn't work. What did he tell you? What's the evidence (Hebrew: re'aya) that he gave you?
78.	Joshua	He doesn't have any evidence yet
79.	Abraham	He does have evidence. That's the evidence he's giving you!
80.	Yehuda	No… That's not evidence
81.	Abraham	Cause these- these expressions actually tell you that any number that you put instead of the expression will give you zero

Continuing the argument while the teacher turned to answer Yehuda, Abraham tried as best as he could to convince Joshua. He said: “Here he’s giving you an expression. That means that anything that you put instead of the expression will give you the same thing". Several more turns occurred between Joshua and Abraham, mainly stating the same challenge and response. However, Joshua remained unconvinced and the episode ended with the teacher moving to show a proof that was more intelligible to the students (the “completing the squares” proof).

ANALYSIS AND DISCUSSION

In analysing the above episode, we first wish to demonstrate it as a hybrid discourse with resources in the ultra-orthodox Talmudic culture and the pre-academic mathematics classroom culture. We then conclude in discussing some of the affordances and problems the hybrid discourse presented the students with, focusing on Joshua and Abraham.

Hybridity

The interactional routines consisted of students actively seeking clarifications both from the teacher, and more importantly, from each other (as in “what’s the evidence he’s giving you?” or “that’s what he’s telling you”). This happened mostly in the last segment, right after the *structure of interactional routines* had changed and Abraham and Joshua were discussing the problem between themselves in the back while the teacher was conversing with Yehuda in the front. In Havruta studies, the most common form of debate is that of using the text to prove one’s point and argue (by challenging or rebutting) against the Havruta peer’s claims. In this episode, we believe the same pattern occurred, with the difference being that here the *teacher’s talk* was serving as the “text” over which students were arguing.

Another hint for the growing dominancy of the Talmud discourse at that point of the lesson was the usage of the key word “re’aya” (evidence), a common notion in the Talmudic studies and debates. This term was inserted into the debate by Abraham and immediately taken up by Joshua. The teacher, being an outsider to the ultra-orthodox world, was not used to this word, neither in the daily nor the mathematical context.

The main key-word the teacher introduced was taken from the mathematical discourse: “counter-example”. This was used to introduce the routine of refuting a mathematical claim. Joshua willingly adopted this new key word, yet he used it in a way that was not intended by the teacher. He repeatedly claimed that Abraham (and thus the teacher) would only be correct "As long as there’s no counter example" [71]. In order to endorse the proof presented by the teacher, one had to realize why there could *not* be any counter-example in that case. In other words, one would have to accept the function that letters (or algebraic symbols) and algebraic manipulations had as generalization tools, and that using a letter instead of a number renders the search for numerical counter-examples to be unnecessary.

Discursive Constraints – the case of Joshua

Joshua did not make any signs of accepting this discursive rule of proving by using algebraic notation. The reason for that might be found in the drifting of Talmudic *authority structure* into classroom discourse. In contrast to Abraham, who at least partially relied on the traditional mathematics classroom authority structure (teacher as arbitrator of truth), Joshua interpreted the teacher's proposal as an object for debate. He was therefore unwilling to "suspend his disbelief" (Ben-Zvi & Sfard, 2007) enough to take the teacher's claim under serious consideration. Perhaps a different conception of the teacher's authority, less Talmudic and more mathematical, would have moved Joshua's focus at that point from interpersonal to intrapersonal activity in order to examine this new idea that the teacher and Abraham were suggesting.

Discursive Possibilities – the case of Abraham

Unlike to Joshua, we claim that the hybrid discourse seen in this episode provided Abraham unique opportunities for learning. Such opportunities would not be available neither in a traditional learning setting where the instruction is strictly teacher-led (in IRE fashion) nor in settings that are mostly student centered (as in small group problem solving). Abraham's mathematical claims developed first through an IRE interaction with the teacher, where he achieved the important realization that there was a deductive way of proving a formula, irrespective of the empirical trials of checking its truth value. However, the intensive "Havruta" episode with Joshua provided the opportunity to *elaborate* and restate his newly acquired narrative in ways that wouldn't have been possible had the conversation remained solely between Abraham and the teacher. Joshua was coming up with questions and challenges that the teacher, who was the one who offered the solution to begin with, was unlikely to present to Abraham.

CONCLUSION

Lest we be misunderstood, we wish to clarify that we are *not* making any claims about ultra-orthodox Jews' general propensity to neither succeed nor fail in mathematics. Neither are we making general claims about the affordances of a Talmudic background for the study of mathematics. Further research would be needed for making such claims. Rather, the case brought here shows an example of constructing a hybrid discourse from different cultures. The study further illustrates that cultural differences can both lever and hinder the learning of mathematics.

We believe this is an essential step in the derivation of pedagogical implementations. For instance, the insights gleaned from this study can assist teachers of the ultra-orthodox population by raising their awareness to the affordances and obstacles that the Talmudic background may provide. More generally, we believe that such examination of hybrid discourses in the mathematics classroom is a fruitful path for educators wishing to integrate students from diverse cultural backgrounds.

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