ZOOMING IN AND OUT - ASSESSING EXPLORATIVE INSTRUCTION THROUGH THREE LENSES

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We examine the implementation of an identical task by two middle-school teachers. Both teachers underwent short professional development training according to Smith and Stein's (2011) 'Five Practices for Orchestrating Productive Mathematics Discussions'. The different levels of implementation are examined through three analytical lenses: Instructional Quality Assessment tool, Accountable Talk coding, and commognitive analysis. Each of these three lenses provides a different resolution of the opportunities for explorative participation students had in the two classrooms. We discuss implications both in terms of benefits for research and in terms of usefulness for teacher training.

Instruction that supports deep, meaningful learning is multifaceted: it involves both certain mathematical activities, as well as a particular social structure to support it. As such, it provides a challenge for any researcher who wishes to examine the extent to which such instruction is implemented in the classroom. The need for such an examination has become urgent in the face of ever-growing attempts to improve mathematics instruction. Most assessments of effective teaching are usually done using a scale designed to measure certain "best practices" (e.g. IQA, Boston, 2012). The use of a single scale may be unhelpful for teachers wishing to improve their instruction. It may either be too general, leaving teachers wondering what to change, or too specific, neglecting the whole picture of the lesson. We suggest to take a different approach: we first define the type of teaching we wish to see in the classroom – explorative instruction. Then, we examine what different analytical tools may tell us about the extent to which this instruction was observed in the classroom.

THEORETICAL BACKGROUND

We define explorative instruction as instruction that supports explorative participation in mathematical learning. Explorative participation (Sfard & Lavie, 2005) is participation for the sake of producing mathematical narratives to solve problems or to describe the world. Such participation is contrasted to *ritual* participation, which main goal is pleasing others and which is characterized by rigid rule following and endorsement of results as "correct" according to external authority. Explorative participation is linked more broadly to the view of mathematical learning as the process by which students gradually become able to communicate about *mathematical objects*. These discursive objects are a result of the "saming" of different *realizations* (Sfard, 2008). For example, connecting between different *visual mediators* such as: graphs, tables and algebraic expressions, is central to the "saming" of the function object. A mathematical object can be visualized as a "realization tree" where complex objects are made of simpler ones.

Given this view of explorative participation, instruction that supports it is characterized by several features. First, it provides tasks that afford multiple opportunities for "saming" different realizations, producing narratives based on different routines and enacting mathematical meta-rules such as conjecturing and proving. In the words of Stein, Grover and Henningsen (1996), such tasks are "cognitively demanding" (p. 461). Second, explorative participation can benefit from instruction that constructs certain *participant frameworks* in the classroom (O'Connor & Michaels, 1993). Such participant frameworks allocate students and teachers appropriate roles and duties to carry out the construction of mathematical narratives, in the face of the initial uneven status where students are less experienced than the teacher in doing so. Accountable Talk (Resnick, Michaels, & O'Connor, 2010) provides a set of talk moves that can create such participation frameworks. In particular, it suggests talk moves for holding students accountable to each other (the community) and to rigorous reasoning. Both types of accountability are important for explorative participation. Accountability to reasoning encourages building narratives based on formerly established narratives; accountability to the community moves the authority structure from being solely based on the teacher, to being more equitably divided between him/her and the students.

In the context of a professional program aimed at offering practical tools for teachers to improve their instruction, we thus asked: to what extent did teachers succeed in giving students' opportunities for explorative participation? This question was divided into the following sub-questions: (1) Were teachers able to maintain the cognitive demand of a task? (2) To what extent did teachers encourage the accountability for reasoning and for the community during the lesson? (3) To what extent did teachers give students opportunities for saming different realizations of mathematical objects?

METHOD

Participants included four teachers of 7th and 8th grade mathematics. The teachers participated in PD training sessions that introduced the main components of the *5 Practices for orchestrating productive mathematics discussions* or 5Ps (Smith & Stein, 2011). The 5Ps framework suggests a set of instructional practices that support the maintenance of high-cognitive demand of a task. These include anticipating students' responses, monitoring their work, selecting solutions to be presented to the whole classroom, sequencing these solutions, and connecting between them. By giving teachers a road-map of steps that they can prepare in advance and during whole-class discussions, these practices have the potential for helping teachers to more effectively orchestrate discussions that are both responsive to students' emerging understandings and emphasize important mathematical ides. As part of the PD, the teachers were asked to implement a lesson prepared according to the 5Ps. They were asked to give their students an identical task, *the Hexagon Task* which asks students to describe the perimeter of a general "train" in a pattern of hexagon "trains" (See Figure 1):

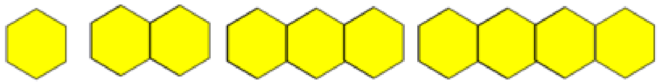

Figure 1: The Hexagons Pattern

The task was used since it had previously been rated as a high cognitive demand task that is productive for teachers' initial attempts to implement the 5Ps (Heyd-Metzuyanim, Smith, Bill, & Resnick, 2016). We observed, video recorded, and transcribed all lessons. In addition, the teachers were interviewed before and after the lessons, and the lesson planning sessions were recorded.

Data Analysis. According to the above explained conceptualization of explorative instruction, we used three analytical tools to examine: Cognitive demand, Accountable Talk, and opportunities for saming different realizations of mathematical objects.

Cognitive Demand: Measuring the general level of implementation of the task was done based on Implementation rubric (AR2) of the Instructional Quality Assessment tool (IQA) (Boston, 2012). This rubric evaluates the cognitive demand of the lesson based on an observation of the recorded lesson. The rubric includes a scale from 1 to 4 where 1 means students engage only in rote memorization and producing facts, 2 means they engage in the application of procedures explicitly taught, 3 means cognitive demand is not lowered but mathematical reasoning is not sufficiently explicated, and 4 means full maintenance of cognitive demand.

Accountable Talk: For achieving a higher resolution of the lesson, and in particular, the participant framework supported in it, we used the Accountable Talk coding scheme (AT) (Resnick et al., 2010). This scheme codes classroom transcriptions on a line-by-line basis. It includes eight codes for teacher moves, where four codes measure accountability to reasoning and knowledge (press for reasoning, challenge, say more and revoice) and four codes measure accountability to the community (add-on, restate, agree/disagree, and solicit additional viewpoints). These moves track the number of teachers' attempts to make students' thinking public, help students reason mathematically, and hold them responsible for attending to the reasoning of others. In addition, the scheme codes students' moves (students' agree/ disagree, students' justification, students' press for reasoning and students' challenge). The two authors achieved 84% agreement on 50% of the data reported in this paper.

Opportunities for objectification: For examining the opportunities for exploring mathematical objects given to students during the lessons we used the Realization Tree Assessment tool (RTA) (Weingarden, Heyd-Metzuyanim, & Nachlieli, 2017). The RTA can be used both to assess the potential of the task to afford the "saming" of different realizations of a mathematical object, as well as the way in which these opportunities actually play out during implementation. It depicts the different realizations of a mathematical object as nodes in a "tree" and then uses different shades to signify who articulated the realization – the teacher or students.

Close-up analysis of opportunities for objectification: Excerpts that have been found to be particularly telling during the scanning of the lesson for RTA analysis were examined on a close-up word-by-word resolution to determine the precise moves used by the teacher to encourage students to articulate certain mathematical narratives. In particular, we looked for teacher questions that encouraged connections between different realizations of mathematical objects and for narratives raised by students and taken up or missed by the teacher.

FINDINGS

Due to space limitations, we restrict our exemplification of the analysis to two teachers who implemented the same task in different ways: Dani, a 7th grade teacher with 3 years of experience and Sivan, an 8th grade teacher with 2 years of experience.

Maintaining cognitive demand during implementation

In Dani's lesson, cognitive demand was maintained and scored at the highest level (4). Scoring was based on observing that Dani did not lead the students towards any particular solution; multiple solutions were found and presented by the students; solutions were linked to each other both by the teacher and by the students; and there was no proceduralization of the task. In contrast, Sivan's implementation was scored as a 2. This, since she led students toward a particular solution (4x+2) that was not exemplified through the visual Hexagon's representation, connections were not made with other algebraic expressions, and students seemed to be well rehearsed in producing a table, algebraic expression from it and a graph of that expression.

Accountable talk

Interestingly, the two lessons were quite similar in terms of Accountability to Reasoning as a whole, that is, aggregating "press", "challenge" ,"say more" and "revoice" moves (Dani N=36, Sivan N=33). However, Dani's talk included more "press for reasoning" (Dani N=23, Sivan N=14) requesting students to justify their claims. In accordance, student justifications in Dani's lesson were higher than in Sivan's (Dani 22, Sivan 11). Most important, moves encouraging Accountability to the Community were much higher in Dani's lesson (12) than in Sivan's lesson (1). Students' talk aligned with these different demands. In Dani's lesson, there were 20 instances of students' agreement/disagreement with their friends mathematical narratives, while in Sivan's lesson there were no such instances (0). Thus, the AT counts point to the participation framework in Dani's classroom being more conducive for students' authority to propose mathematical narratives than Sivan's lesson was.

Opportunities for saming different realizations of mathematical objects

Sivan's RTA (Figure 2a) and Dani's RTA (Figure 2b) show that as a whole, the classroom discussion made different realizations of the object "perimeter of a general train" accessible to students.

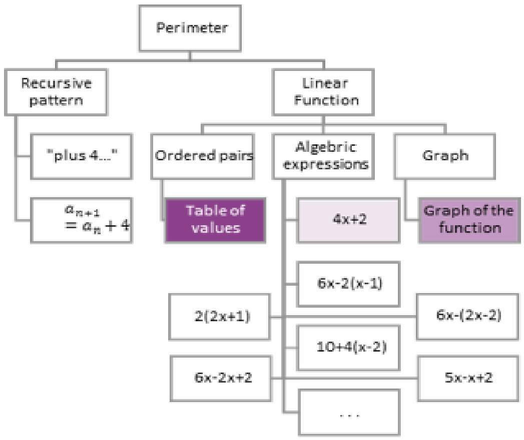

Figure 2a: Sivan's RTA

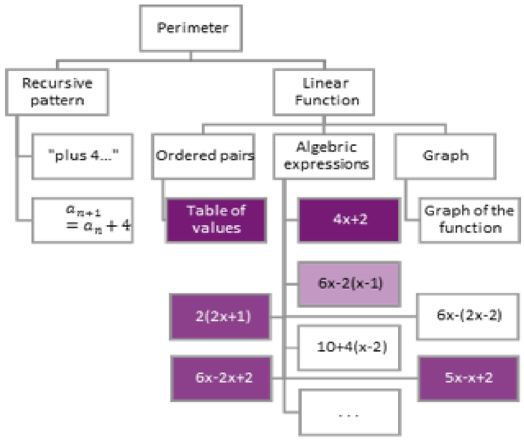

Figure 2b: Dani's RTA

4	The student's explanation was complete
3	The student's explanation was not complete. The teacher explicated the idea
2	The student did not articulate the realization, but the teacher did
1	The realization was mentioned, but neither the student nor the teacher explained it fully
0	The realization was not mentioned, but was hypothesized to be relevant to the task

Dani's RTA shows that his classroom discussion included more narratives about the perimeter object, especially of the algebraic type, than Sivan's discussion. Moreover, the dark colours of Dani's RTA show that his students were more active in forming the narratives around these different realizations, while Sivan was mostly responsible for the narratives in her lesson, eliciting them from the students or simply presenting them on the board. Though Sivan did refer to different realizations of the perimeter (table, algebraic expression and graph), the 'branch' explored in Dani's lesson provided an excellent opportunity to connect different algebraic expressions to a single visual mediator (the Hexagons), as there are various different algebraic expressions that express the desired perimeter, based in how the hexagon-sides are counted and connected to the perimeter (referring to external sides only, all hexagon sides and then taking away inner sides, etc.). In contrast, Sivan took the Hexagons visual mediator only as a starting point and directed most of the discussion to realizations of a linear function, which were not directly connected to the Hexagon's mediator. In that sense, Dani utilized the visual mediator to a much greater extent.

Close-up analysis of instructional talk supporting the saming of different realizations

The wide view of the mathematical narratives raised in the lesson, given by the RTA, show that Sivan's lesson was more limited in its affordance for saming different realizations. However, this snapshot view is limited in its ability to lend any explanation as to *why* this restriction occurred, given the multiple affordances of the task. In fact, Sivan herself was aware of this restriction but did not know to explain why it occurred. In her post-lesson interview, she said: "If there was even one student who found another (expression), I would have asked him immediately to show it to everyone. But they just all came up with 4x+2". However, the examination of students' discourse revealed otherwise. See for example an instance where a student was trying to explain why he came up with the expression 4x+2:

51	Eitan	The four is like the difference between each perimeter. Let's say, train number one has 6 and train number two has 10.
52	T	O.K. I'll write like this, train number 1 is 6, train number 2 is 10… (writes up a table of values) and here, you saw what? (Draws arcs between the table rows)
53	Eitan	Plus four
56	T	O.K. So I found this (pointing to 4x) four… What's the '2'?

Taking Eitan's explanation that related to the visual mediator ("train number one", etc. [51]), the teacher transformed this narrative into a well-rehearsed visual mediator of a table with "arcs". The fact that this was well known to the students could be seen in the fact that Eitan easily fit the appropriate "blank" in the teacher's prompt "you saw what?"[52] with a "plus four" [53]. The teacher's next question, "What's the '2'?" [56] was more difficult for Eitan to see from the table. Thus, he hesitated while another student, Orit, asked to explain.

61	Orit	These two sides that seem to be connected and that we, like, don't consider them in the drawing, there are always these two sides that get connected to form one side… so they get reduced from the perimeter, so that is the '2'.
62	T	So you say there are two sides connected, so instead of taking them off, you add them? … Why not to do minus two? Aren't you taking them off?

The teacher's question [62], which pertained to whether the '2' should be added or removed from 4x, reveals that she was not seeing the visual mediator of the perimeter the same way Orit was suggesting. Thus, she missed a visualization that could have led to another narrative: 6x-2(x-1) which describes the visualization of all hexagon sides counted, then the double inner sides taken away. While Orit pondered around the teacher's question, Eitan offered an explanation according to the tables of values:

65	Eitan	To reach 4 (probably means 6) we had to add two then we added the two.
66	T	Oh. So you just substituted, you saw it's true. So is it always true?
67	Eitan	Yes.

With this, Sivan left the issue of where the '4' and '2' "come from". Once declaring the correctness of the 4x + 2 expression, she invited a student to draw on the board a graph depicting y=4x+2. The drawing of the graph was done according the algebraic expression by locating one point on the Cartesian plane according to the intercept ((0,2)), and another point according to the rate of change ((1,4)) and stretching a line between them. In fact, the only reference back to the Hexagon's occurred when Sivan asked the students "so when I have 0 hexagons, their perimeter is 2?!" which led to a discussion of the domain and range of the function but was not implicated on the graph itself. In contrast, Dani offered many opportunities for students to connect between the different realizations, including the Hexagon's perimeter, table of values and algebraic expressions. He pressed for these connections consistently. For example, when a student suggested the expression 2(2x+1) Dani said: "O.K., O.K., but why is this true, in relation to the trains?" and when a student suggested another expression (6x-2x+2), he asked: "but how do you explain it with respect to the drawing?" In addition to this insistence on relating expressions to the Hexagon's perimeter, Dani also invited other realizations. He asked a student, who he had seen working on a table, to present his work. After the student's presentation, he revoiced it:

> "Tom built a table… Tom built the placement and the number … once he did that, what did he actually do? He detached himself from the trains. He is no longer thinking about the trains. He only looks at the numbers and tries to find a pattern in the numbers, alright?"

By revoicing Tom's solution, Dani not only explained it in terms of "placement" and "number" (meaning number of sides), he also clarified that the table *could* have been related to the Hexagons ("he detached himself from the trains"). In addition to encouraging connections to the visual mediator, Dani also asked students to explain why 6x-2(x-1) was "true" "algebraically", encouraging students to use algebraic manipulation routines to prove that 6x-2(x-1) = 4x+2.

CONCLUSION AND DISCUSSION

Our goal in this paper was to examine the instructional practices of teachers introduced through a short PD to instructional practices that can support explorative participation. The two contrasting cases afforded us the opportunity to better understand what each of our three different analytical lenses shows about the lesson. These three lenses cohere in showing that Dani's lesson gave more opportunities for explorative participation than that of Sivan. The IQA implementation measure gave the first, rather coarse indication, that Dani maintained the cognitive demand of the task while Sivan lowered it; AT coding showed that Dani encouraged more accountability to the community and to reasoning than Sivan and that his students were more accountable for the community; the RTA showed that Dani's lesson included more mathematical narratives about different realizations of the "general perimeter" object (which in later grades would be called "the general term" of a sequence) than Sivan's lesson. Finally, the close-up analysis, of specific words and sentences, showed that Sivan missed some potential narratives relating the algebraic expressions to the Hexagon trains, and did not make attempts to links between representations and connect them to the visual mediators, while Dani persistently pressed for such links.

The tools we used in this research are useful not only for their varying resolutions and foci, they also have different potential to help teachers improve their instruction. Measures such as the IQA's general implantation rubric are good for assessing the successful implementation of a task. However, they are less effective in showing teachers where they actually could do things differently. Accountable Talk moves provide a more useful tool, in our view, since they give teachers specific suggestions for what they can say in the classroom and give them a clear image of what they need to look for in students' talk. Similarly, but focusing on the mathematical content, commognitive analysis can show teachers what specific talk moves may elicit the saming of different realizations. This, in addition to the RTA which can help teachers map the narratives they wish to elicit from students based on the mathematical task at hand. Our multifocal lens approach thus holds promise both for research purposes and for teacher training purposes. It is, however, only in its initial stages. Therefore, further studies will be needed to establish its coherence and usefulness.

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