

# TEACHERS' DISCOURSE ON STUDENTS' CONCEPTUAL UNDERSTANDING AND STRUGGLE

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*We employ a communicational lens on the discourse of elementary mathematics teachers, asked to identify themselves with relation to vignettes describing four teaching types: high/low student struggle and high/low attention to concepts. Our goal is to examine the narratives that support low struggle or low attention to concepts. Data included interviews with four experienced elementary school teachers. Findings show that teachers had a coherent story for why they adopted or rejected each teaching type and that support of other-than-optimal teaching types was related to their conceptualization of "learning with understanding" as well as the ways in which they identify students of different "abilities".*

## BACKGROUND

In a review of links between teaching practices and students' learning, Hiebert and Grouws (2007) pointed to the importance of two aspects in teaching: explicit attention to concepts (EAC), defined as "the public noting of connections among mathematical facts, procedures and ideas" (p. 383) and students' opportunity to struggle (SOS), that is: "students' expending effort to make sense of mathematics, to figure something out that is not immediately apparent" (p. 387). Optimal teaching, they claimed, combines both EAC **and** SOS. Schoenfeld (2014) also concludes that such instruction, which lends students authority, as well as exposes them to important mathematical ideas, is the best for achieving robust learning.

Yet studies show that such teaching, despite decades of curricular reform and professional development attempts, is still pretty rare (Resnick, 2015). Moreover, changing teachers' practice may prove to be a long and difficult process (Guskey, 2002). In the present study, we offer to view this as a result of teaching practices being a part of a *pedagogical* discourse or discourse about teaching and learning. Similar to any other discourse (Sfard, 2008), it is made of certain key-words, narratives, and meta-rules. These dictate *what* to teach students, *how* to teach them and, often not talked about but still very important, *who* can learn (or not learn). This view is anchored in Sfard's (2008) view of mathematizing as participating in a discourse about mathematical objects thus the pedagogical (the *how* and for *whom*) is closely intertwined with the *what*.

Participation in pedagogical discourse is very much a matter of constructing a certain *identity* of oneself as a teacher (Goos, 2005). Thus, the story a teacher tells about herself is constructed on the web of narratives she endorses about teaching and learning. Examining the ways in which teachers identify themselves with relation to certain prototypical practices holds the potential to unearth the web of narratives teachers endorse as part of their “pedagogical frame” – the set of meta-rules determining what is effective teaching for them.

Based on Hiebert and Grouws’ (2007) work, Stein and her colleagues (Stein et al., 2017) have come up with a framework that divides teaching into relatively simple “types”. These are named “quadrants” and typify teaching according to high or low levels of the two aspects identified as most important for students’ learning: explicit attention to concepts, and students’ opportunities for struggle (see Figure 1). Stein et al (2016) developed a survey based on vignettes of a “typical lesson” of these four quadrants.

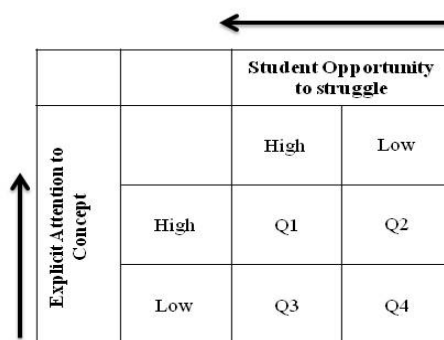


Figure 1: The Teaching Quadrants (according to Stein et al., 2017)

These vignettes, which present relatively simple but sufficiently informative typification of the four teaching approaches, offer teachers an opportunity to identify with alternative forms to the “optimal” view of high EAC/high Struggle. The understanding of these alternative approaches is important for disrupting them and moving teachers towards instruction that is high both in EAC and in student struggle. Our question in this research was thus: what may the discourse of teachers around vignettes of “typical quadrant teaching” reveal about teachers’ identity and their reasons for adopting or rejecting high EAC and high SOS?

## METHOD

Since our goal was to compare and contrast narratives about teaching, we chose four teachers that the first author, from her professional role as district instructor of mathematics, knew to be quite different in their teaching practices. All teachers held

teaching certificates ranging from a B.Ed to M.Ed and had experience ranging from 11 to 25 years of teaching mathematics. They were asked to answer Stein and her colleagues' survey. The survey includes 6 vignettes depicting different types of teaching practices. Each vignette is constructed to describe a "typical" quadrant, without, of course, hinting which teacher is better or that there are, in fact, such "quadrants" underlying the vignettes. Following is a short description of each of the vignettes (the originals are around half a page):

All four vignettes describe a lesson dealing with the same subject: connecting fractions, decimals and percents. The Q1, Q2, and Q3 teachers all use a similar task, which affords connections between an area diagram, fractions, decimals and percents.

The Q1 teacher - presents the topic for the lesson, then hands out the task for students to work on in groups. She then walks around and assists with specific questions tailored to advance students' thinking. At the end of the lesson she invites two students who have found different solution paths to present and discuss their work. She then draws students' attention to the equivalence of the three representations (fraction, decimal and percent) as seen in the different diagrams.

The Q2 teacher – starts the lesson similarly to Q1 but after most students apply one solution, she points to the equivalence of the diagram and the fraction, then elicits from students the equivalence to percents. She concludes by explaining the meaning of equivalence.

The Q3 Teacher - starts the lesson similarly to Q1 and walks around students monitoring their work. When students make mistakes, she asks them to: "think harder" but does not guide their thinking. At the end of the lesson she invites a group to present their correct solution. Connections between fractions, decimals and percents are not made explicit at the end of the lesson.

The Q4 teacher chooses a different task, which would provide opportunity for targeted practice on an efficient procedure for converting fractions, decimals and percents. She demonstrates the procedure and then gives students similar tasks to work on individually.

After teachers completed the survey, they were individually interviewed on it by the first author. Each semi-structured interview lasted around 30-40 minutes and was designed to elicit teachers discourse around key pedagogical words such as "conceptual/procedural understanding" as well as teachers' identity narratives in relation to the vignettes.

Interviews were fully transcribed and first analysed in search of common issues and statements. Next, we paid attention to particular sentences and word use, for example, around “understanding” and “students”.

## FINDINGS

Only one of the teachers identified herself with the Q1 teacher. The rest identified themselves as either between Q1-Q3, Q2, or “eclectic”. In what follows, we first describe the self-identifications of the teachers and their relation to their definition of “understanding”. We then move to present some commonalities in their discourse about students.

### **Hadar: Identifying with Q3**

Hadar hesitated at first between the Q1 and Q3 vignettes. She said: “I feel that at work I zig-zag between the two”, explaining: “one (Q1) gave complete freedom to students. Counted on them. The other moderated them a bit”. Eventually, she leaned towards Q3, explaining that that the Q1 teacher “gives too many hints”. She, in contrast, likes “that they (the students) struggle themselves and then I create a conflict, and only facilitate the discourse”. In that sense, it is clear that Hadar picked up the vignettes’ depiction of the Q3 teacher as letting students “struggle themselves”. However, she did not appreciate the Q1’s teacher explicit attention to concepts. Rather, she interpreted that as “giving too many hints”. We found Hadar’s discourse around “conceptual understanding” linked to this neglect of EAC:

Conceptual understanding is when I know what can belong to a concept and what does not belong to it. Like, a square, I can define what is a square and what is not a square, so I have a definition of the square concept.

We found this to be a rather constrained conceptualization of conceptual understanding. It does not mention relations between objects, between procedures or between different representations (graphical, numerical, etc.). Hadar was missing in the Q1 vignette an indication of “cognitive conflict”. Thus, for her, “understanding” seemed to be only facilitated by “conflict” not by other means of relating between different representations and procedures.

### **Hila: Identifying with Q2**

Hila identified herself with the vignette of the Q2 teacher. She explained:

She works in a gradual way. She doesn’t send them straight into the lion’s den ... (she) works with them step by step. She makes them understand together the complex task while not giving up on the difficulty of the task, like Sharon (Q4 teacher).

Even from this short excerpt, one can see Hila views student struggle very differently than Hadar. For her, such struggle is a threatening experience (“lion’s den”) that students should be protected from. “Student understanding” is achieved through “working with them together, step by step”. What such “understanding” means for Hila is revealed in her example of how she ensures her students will “understand”:

For instance, when they study long division. So it’s important that they learn the way (procedure). And understand why and how to do each step. I connect it to DMSB (explains a Hebrew mnemonic for memorizing the steps of long division)... That way they have a good understanding of long division.

Thus, Hila equates “good understanding” with the correct memorization and execution of procedures. She does not make links to mathematical objects or to connections between routines.

### **Dana: I’m a little bit of everything**

Dana insisted she could not identify herself with any particular type of teaching. She explained:

My lessons really change a lot from lesson to lesson, according to what I feel. I really really think that I use everything. I can use the methods of Nitza (Q1), needing to give directions and hints, and there’s a lesson that I would actually use Sharon’s (Q4) technique. And there are periods, or days, or a year, that I would act otherwise.

The insistence of Dana that she could not identify with any types of teaching represents, in our view, a narrative of itself: by claiming she uses eclectically different “methods”, Dana resists the idea that she should adopt a certain type of coherent instructional practice. She continues:

So to say that I only do technique, that’s the least correct. To say that I only use tasks that are explored independently, that’s totally incorrect, and to say that I facilitate all the time – also incorrect. In short, it really really depends.

Though she rejects identifying herself with any specific teaching style, the ways in which each of the vignettes is interpreted by Dana is pretty clear: Q1 “gives directions”, Q2, “constantly facilitates”, Q3 gives “tasks for exploring alone”, and Q4 “does only technique”. These rather shallow labels clarify that for Dana, none of the vignettes signifies a coherent teaching approach. Further explaining her choice of “method” Dana explains:

When I open a subject and introduce students to a subject, I give them (the students) a task, and I say ‘take your time, and work alone, and inquire, and check’. It could take one day or two or three or even a week, and (I tell them) ‘explore on your own’ ... I can open with such tasks that will lead them to insights, but at a certain point, you turn to technique and you direct (them).

Thus, for Dana, “insights” are not connected with “technique”. “Independent exploration” is reserved for the slow process of “gaining insights” and is the luxury of “beginning a subject”. Once that luxury is over, she has to step in and “teach the technique”. This is connected, again, to her conceptualization of “understanding”. Explaining her insistence on “understanding”, especially in lower grades, she gives an example:

I am now starting (with my 2<sup>nd</sup> graders) the numbers in the domain of 100. And I’m supposed to start in a short while long addition and subtraction. And since September I’ve been working on Digi (base ten) blocks, on units and tens, on composition, and on tens. And really put effort into their understanding. What composition actually means.

Thus, Dana mostly equates understanding with a slow process whereby students engage with manipulatives to be able to “understand”, and eventually follow a certain procedure (composition). She never mentions connections between mathematical objects or a relation to a wider web of mathematical ideas.

### **Nira: Identifying with Q1**

Nira expressed her self-identification very clearly:

I’m like Nitza (Q1), period. I give them challenging tasks, and then assist as much as needed and according to the difficulty that arises. And I give space for independent inquiry.... I don’t tell too much and I don’t give students unrealistic work.

Thus, similar to the other teachers, Nira located herself between two extremes: “too much telling” and “unrealistic work” (or challenge). However, unlike the other teachers, Nira had a pretty clear vision of how this type of instruction connects to “understanding”, and in particular, to understanding of low-achieving students:

She (Q1 teacher) can help them (the students). She can take them from the place they’re at and make them fully understand, deeply, any subject. She will work on connections to other subjects, and on different representations.

Nira’s clear view of how students “independent inquiry” can lead to “understanding” was connected to her description of “conceptual understanding” which was, by far, the richest we received from our interviewees:

(Conceptual understanding) is understanding the subject in any form it can be represented and also the relations between the concepts in that subject, ... For instance, multiplication – understanding the relation to the area model, understanding the relation to repeated addition, understanding that it also belongs to proportional reasoning, and that it can be described by repeated jumps on the number-line.

### **Teachers’ discourse about students**

There was one interesting commonality to all three teachers, except Nira: they all differentiated between their practices with students who have “different abilities”. Importantly, this issue was not raised by the interviewer, neither was it a part of the survey. Hila (Q2) talked about matching her regular instruction to the abilities of the “middle group” and about working differently with “low ability” (or “weak”) students, who “needed something more technical”. A similar narrative was told by Dana (Eclectic), who said: “If it’s a student with difficulties that I know that has no choice, then I work on the technique”.

Hadar (Q3) did not explicitly label students as being of a particular type, yet she still referred to students that deserve “other” types of instruction:

There are kids that I know, for example, that showing them the algorithm, or explaining the procedure, the solution... I know that they won’t succeed in understanding, and I do want them to know, so I use it (Q4 instruction).

Given the issue raised by the three interviewees, about differentiating teaching according to students’ “abilities”, we went back to Nira (Q1) and asked her how she would teach “students with difficulties”. Nira reacted with some puzzlement to the question, answering immediately: “(I teach) regularly, why?” When hearing that other teachers thought it was an important factor, she added:

Look, for students with difficulties, it’s important to give scaffolds, manipulatives or anything that would help them work on the same tasks that are learned in the classroom, so they don’t feel behind.

Still, she insisted that the Q1 teaching is the best for these types of students and in fact, “can help them the most”.

## SUMMARY AND CONCLUSIONS

Our goal in this study was to expose the narratives underlying choices of teachers to identify with particular types of teaching, according to high/low EAC and high/low SOS. The findings show that each teacher had a coherent set of narratives for explaining her choice (or avoiding the choice) of a particular teaching type. Thus, the vignettes were highly effective in eliciting teachers’ identity narratives and in helping them reflect on their teaching practice.

Our findings also point to a possible relationship between teachers’ choice of which vignette to identify with, and their discourse on students’ “understanding”. Except for Nira (Q1), the three teachers’ discourse about “conceptual understanding”, or “understanding” more generally, was quite limited, and mostly referred to being able to follow and explain a given procedure correctly. There also seemed to be a disconnect between building on what students already know (seen in words such as

“inquiry” and “gaining insights”) and having students carry out mathematical procedures (“the technique”). These differentiations went often together with the identification of who can “understand” and who can “only do the technique”.

These findings, as initial and embryonic as they are, point to the possibility of there being a relationship between narratives about mathematics – being a set of rules to be followed or being an interconnected web of relations between mathematical objects – and narratives about students. In other words, it seems the narrative that certain students are “simply not able to understand so they need do the technique” is easier to endorse when “understanding” and “technique” are differentiated.

Unfortunately, this pedagogical discourse may be, in part, responsible for the construction of learning difficulties to begin with. Previous research (e.g. Heyd-Metzuyanin, 2013) has shown how a teacher and a low-achieving student “co-construct” the students difficulties by both sticking to ritual rule following, in the face of the students’ ever-growing gaps vis-à-vis the curriculum. However, attempts to disrupt the common belief that low-achieving students should engage with cognitively demanding tasks are still rare.

Another insight we gained is that none of the teachers interviewed on the vignettes actually related to the *explicit attention to concepts* in them. In fact, teachers judged the appropriateness of the practices almost solely based on the *struggle* aspect of the story, essentially placing all vignettes on one ‘struggle scale’ (roughly Q3, Q1, Q2 and Q4, from highest to lowest). This finding hints at the ubiquity of teachers’ discourse around students’ struggle (good or bad), at the price of discourse on attention to concepts, or mathematical narratives, more generally. It also echoes Chazan & Ball’s (1999) well-known lament about teachers only being “told not to tell”, while *what* to tell (or not to tell) is not being explicated.

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