Characteristics of students' explorative participation while playing games in middle-school mathematics lessons

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1. Abstract

Educational systems worldwide now encourage and support the transition from traditional teaching to student-centered teaching. Student-centered teaching prompts independent, engaging, hands-on learning, as well as the development of required skills for the 21st century such as critical thinking and working in groups. This study focused on students' face-to-face game-playing in groups during mathematics lessons. Previous work on digital games has suggested that students' achievements, motivation, and attitude towards mathematics improves when playing educational games. Most studies have applied quantitative methods to explore the outcomes of the learning processes during digital games. By contrast, this thesis examined the ways in which students participate in mathematical discussions while playing by constructing a methodological instrument based on the Commognitive framework which centers on the characteristics of explorative participation. Commognitive research is grounded in the socio-cultural approach and views learning as a change in students' participation in mathematical discourse from more ritualistic towards more explorative. This study also examined which characteristics of a game promote or hinder students' explorative participation. To achieve these goals, this study took a close look at five groups of 9th grade students who were videotaped while playing three different mathematical games. The videos were transcribed verbatim. The videos and transcriptions were analyzed by attending to the characteristics of the students' explorative participation, including agentivity (the group's independent engagement and choice of how to do mathematics), bondedness (the way one step in a solution leads to the next), applicability (the use of previous narratives and routines to solve mathematical problems), substantiability (justification of routines that were suggested in groups) and flexibility (solving the same task with different procedures).

The findings show that students' participation was explorative in all games in terms of their agentivity, bondedness and applicability. There were different modes of participation for each characteristic (termed types here). Students' agentivity in groups was dominant in all groups and games, as evidenced in four different types of participation (planning, clarifying, producing and executing mathematical narratives). Surprisingly, two types of applicability had to do with applying a new procedure while playing. The data suggest that there was a structure of global bondedness that included sub-routines for each global step in the global routine. Substantiability was only evidenced when students disagreed or had to clarify a mathematical routine or narrative. Flexibility was rare and was not promoted by the game designs. Further analysis of the game designs showed that some components of the games

encouraged group agentivity (the number of players and shared cards), bondedness (executing moves of more than one step) and applicability (based on familiar narratives and routines) while others hindered group flexibility (computer feedback) and did not promote substantiability.

This study makes a practical contribution to teachers who would like to introduce mathematical games to the classroom. By doing so, they can promote explorative participation but will have to actively encourage flexibility and substantiation. This study contributes methodologically by providing a measure of students' participation while playing face-to- face games, and game design.

2. Introduction

Educational systems worldwide now encourage and support the transition from traditional teacher-centered teaching to student-centered teaching (Brown, 2003; Mascolo, 2009). Teacher-centered approaches conceptualize teaching as the dissemination of knowledge, and emphasize formalized mathematics, which is presented as a collection of facts and procedures (Gregg, 1995). Students in teacher-centered classes are mainly expected to replicate these procedures (Brown, Cooney & Jones, 1990). Student-centered approaches to teaching, on the other hand, are based on the Constructivist paradigm that goes back to Dewey and Vygotsky (Agrahari, 2016; Deboer, 2002; Din & Wheatley, 2007). In these classrooms, students are encouraged to develop their mathematical knowledge and conceptual understanding by solving problem situations that challenge their conceptual understanding (Brown, 2003). They are expected to explain and justify their mathematical choices and develop intellectual autonomy (Gregg, 1995).

Game playing in the mathematics classroom is likely to promote student-centered teaching because while playing, students are encouraged to make their own decisions while communicating with others, and to authentically engage in the tasks called for by the game. Studies have shown that young students are more motivated when playing educational games (in different disciplinary fields) and evidence greater achievement (Henry, 1973; Gee, 2003a; Gee, 2003b; Plass et al, 2015). However, studies on educational games have mostly dealt with younger pupils and are based on questionnaires and pre/post-test comparisons. They focus on outcomes rather than how students' participation takes place (Becker, 2010; Gee, 2011). Mathematical games are used in class primarily as an incentive to learning (Bragg, 2006b), a drill, for practice (Lim-Teo, 1991) or as a complementary activity (Bragg, 2006b).

Thus, little is known about how students participate and learn while playing mathematical games.

Although some studies have reported that educational games have certain benefits, mathematics teachers who aim to implement them in class or create a new game for their students are often unsure as to what constitutes a successful game and how they as teachers can evaluate a game before using it in class (Kafai, Franke, Ching, & Shih, 1998; Russo, Russo, & Bragg, 2021; Wiersum, 2012). The main criterion when considering a game design is immersion (Petri, Wangenheim, & Borgatto, 2016); in other words, the extent to which a game is successful in engaging and motivating students. Immersion tends to be applied to digital games (Bragg, 2006a; Petri, Wangenheim, & Borgatto, 2016). Studies have also evaluated the relevancy of the mathematical content to the curriculum, which mathematical skills are required and how these are combined in the game (Laato, Lindberg, Laine, Bui, Brezovszky, Koivunen, & Lehtinen, 2020; Tran & Nguyen, 2020). This study examined the opportunities to participate that a game design and its rules provide to learners, and the nature of their actual characteristics of participation while playing mathematical games.

The commognitive framework (Sfard, 2008), similar to other sociocultural approaches, views learning as becoming a participant in the discourse of a particular community. Learning mathematics is the process by which students gradually become able to communicate about mathematical objects. Commognition distinguishes between two types of students' participation in the mathematics classrooms: ritual participation, during which the learner is focused on performing procedures, often mimicking previously learned ones, and explorative participation during which students aim at producing narratives about mathematical objects on their own (Nachlieli & Tabach, 2019).

As a mathematics teacher in middle school, I have been encouraging game-playing as an inseparable part of my mathematics lessons under the assumption that as the literature indicates, students' engagement with learning mathematics and motivation to solve mathematical problems improves when applying mathematical games in class (Bragg, 2003; Deater-Deckard, Chang, & Evans, 2013; Plass, O'Keefe, Homer, Case, Hayward, Stein & Perlin, 2013). However, not much is known about how students participate while playing. In addition, games differ considerably in terms of the opportunities for participation provided to students. Therefore, the goal of the present study was twofold: to examine how students participate through mathematical game-playing in specific type of games, and to identify the opportunities for explorative participation which can be promoted by the game.

3. Theoretical background

This study examined students' modes of participation in mathematical discourse while playing mathematical games in class and how this participation can be promoted or hindered by games. This chapter first presents the world of games and specifically educational games. Then a detailed section digs deep into the components of educational games as well as the way they can be designed and then evaluated. This is a necessary step for teachers to assess the potential of games to promote specific types of student participation. Finally, I describe the Commognitive framework, the conceptual framework for this study, and outline the main tenets that are pertinent to the current study on game-playing in middle school mathematics classrooms.

3.1 Learning and teaching through games

Play is considered a crucial activity that contributes to child development. Children tend to play as an enjoyable activity in which they choose to participate (Piaget, 1962; Vygotsky,1976; Vygotsky, 1978). Players may find themselves in a quasi-reality where they are free to explore their behavior, ideas, culture, and social norms without negative outcomes in the real world (Van der Poel, 1994). Players are active participants, and when there is more than one player, interactions take place (Inbar and Stoll, 1970). A game is a narrower version of ‘play’ with a clear goal of winning (Wilson, 1985; Boller & Kapp, 2017). Since games are a self-contained unit, a specific game space is created. In this space, students have opportunities to create and choose strategies without being concerned with outcomes in real life (outside the game) (Hays, 2005; Kapp, 2013).

From a socio-cultural point of view, (Vygotsky, 1976; Vygotsky, 1978) playing can promote learning through spontaneous interactions with peers or adults who can enlarge students’ zone of proximal development: “in play it is as though he (the child) were a head taller” (p. 103). The challenge must be within learners’ zone of proximal development to contribute to the learning process (Hamari, Shernoff, Rowe, Coller, Asbell-Clarke, & Edwards, 2016). Teaching through games is also well supported by Constructivist theory since games are natural social activities that encourage children to be involved in their own learning process (Vankúš, 2005). Piaget (1962) pointed out that rules that evoke competition also contribute to negotiation activity and cooperation. Since rules cannot be changed unless all the players agree, children become more aware of others’ points of view and engage in learning that is more cooperative.

Student-centered teaching is grounded in the Constructivist paradigm which claims that a student should learn by experience rather than by merely listening to and mimicking experts. This approach encourages students’ independence to learn from one another actively (Collins & O’Brien, 2003). Teaching young children often includes using educational games since they engage with the learning process and thus create an intrinsic motivation to learn (Bradshaw & Lowenstein, 2007; Bragg, 2003). The interaction with others or with one’s thoughts during the game is the most important component of play. According to Gee (2003b), learning takes place through the social interactions generated by students’ participation in a game. Teaching through games may thus be considered a student-centered teaching approach (Rodkroh, Suwannatthachote & Kaemkate, 2013). The present study followed students’ participation and learning in games that require more than one player (in groups). Though the goal was to win individually, the group space was constituted by the shared mathematical interactions that occurred in the group while playing the mathematical games. Therefore, face-to-face interactions rather than virtual or single-player games were chosen.

Teaching through games exists in a variety of platforms, most of which are digital (Abdul & Felicia, 2015; Ratan & Ritterfeld, 2009). By using games, and more specifically mathematical non-digital games in the classroom, teachers can allocate a specific time and place (turns) for each student (player) to be actively involved in the process of learning mathematics (Friedlander, 1977; Talan, et al., 2020). If the game meets students’ mathematical needs, it becomes much more than a nice drill (Friedlander, Markovits, & Bruckheimer,1988). Playing mathematical games allows students to enhance their mathematical intuition and thinking. It encourages students to develop internal and external dialogues on mathematical ideas (Davies, 1995). Games improve mental calculation abilities, provide an environment in which students generate their own mathematics questions and problems during the game (Parsons, 2008), help to conceptualize mathematical problems, improve students' attitude towards mathematics, motivate students to practice in an enjoyable way, teach mathematical vocabulary and ideas, improve math literacy (Henry, 1973; Siew, Geofrey & Lee, 2016) and improve logical thinking (Orim, Ekonesi & Ekwueme, 2011).

Thus, games lead players to experience a whole new way of engaging in their own learning of mathematics by interacting with others (Way, 2011; Wiersum, 2012).

However, from a pedagogical point of view, little is known about the best way to incorporate games into teaching (McClarty et al, 2012). Some teachers incorporate games as an incentive at the beginning of a lesson or as a break from 'real' learning, rather than as a pedagogical method (Bragg, 2006c; Barzilai & Blau, 2014). Other teachers consider that educational games are time-consuming. They note the time involved in preparing the game, and the time it takes to implement both digital and non-digital games in the classroom (Naik, 2014; Talan, et al., 2020). Some teachers claim that games are mostly for entertainment and for young children (Klofer, Osterweil & Salen, 2009). Teachers are often concerned with class management when incorporating games in class. Some of these challenges have to do with the competitive aspects of game-playing (Bourgonjon, De Grove, De Smet, Van Looy, Soetaert, & Valcke, 2013). A meta-analysis (Chen, Shih, & Law, 2020) that examined the influence of competitive participation in games concluded that competition best contributed to K12 math students in specific type of digital games (the matching type such as puzzles).

Nevertheless, the use of games in education has increased in many subjects including mathematics (Vankúš, 2005). Some studies on mathematical games recommend optimal ways to utilize specific games in class (Bragg, 2012; Taja-on, 2019; Trinter, Brighton, & Moon, 2015; Vondráková, Beremlijski, Litschmannová, & Mařík, 2018). For example, *Steeplechase*, a board game about substituting values in algebraic expressions, was implemented in groups of four where the students were expected to apply mathematical concepts by reverse thinking (Friedlander, 1977). Naturally, many studies on mathematical games tend to describe the game and how to use it in class. The next section looks specifically games designed for educational purposes.

3.2 Educational games

The game-based learning (GBL) approach covers various types of games. GBL promotes using games that create learning environments in which pedagogical objectives are fully integrated with the game. Therefore, participating in the game is tantamount to learning the subject. Games associated with the GBL approach include serious games, educational games, traditional games and instructional games (Noemí & Máximo, 2014; Plass, Homer & Kinzer, 2015).

Serious games were first defined by Abt (1970) as games designed for a primary purpose other than pure entertainment and include both digital and non-digital games. Serious games aim to achieve a range of goals such as training (mainly government training), education and informing the public. The term *educational or didactical game* is a type of serious game designed solely for educational purposes with a more specific educational outcome.

In recent years most educational games have been digital (Gee, 2009; Noemí & Máximo, 2014; Susi & Whitton, 2016; Annetta, 2010), and designed to address a specific content such as mathematics. The basic features of these games include a feedback system that informs the players about their progress towards the goal (Giunti, Baum, Giunta, Plazzotta, Benitez, Gómez, & de Quiros, 2015). There are fewer studies on non-digital games (Talan, Tarık, Yunus Doğan, Veli Batdı, 2020). However, both digital and non-digital educational games are classified as the third generation of educational games that are strongly grounded in Constructivist theory and emphasize the importance of players' social interactions while participating in educational games (Egenfeldt-Nielsen, 2011). Kafai (2006) presented the Instructionist and the Constructivist perspectives. The Instructionist perspective is presented in this section and refers to embedding pedagogical goals in games; for instance, practicing the multiplication table by throwing dice or using a digital game like *Math Blaster*. Constructivists suggest another type of approach where students create their own games (Kafai, 2006).

This study presents three educational games from the instructional perspective. Two are non-digital games and the third is a computer game (on the Desmos platform) with a basic feedback system which reflects the students' win or lose situation at each level. Though the games differ from each other, they can all be considered educational mathematical games. Even within instructional games, there are various definition of educational games. The following section looks at the different definitions of educational games and specifically on their key features.

3.3 Components of educational games as a function of game definitions.

Differences in the definitions of games in the literature point to differences in the components of the games and the nature of the players' interactions. The characteristics of games in this study are presented in chapter 4.1 .They are are based on these definitions, which are reviewed below Some definitions in literature emphasize different facets of games. For instance, Harvey & Bright (1985) considered that the basic elements of games are as follows:

: ”A *game involves a challenge against either a task or an opponent; a game is governed by a definite set of rules; a game is freely engaged in”* (p. 2). Salem, Tekinbaş and Zimerman (2004) defined a game as : “*a system in which players engage in an artificial conflict, defined by rules, that results in a quantifiable outcome*” (p.23). Bragg’s definition (2006a) refers to luck- “…games include elements of skill and/or strategy: Its outcome is not solely based on luck…” (p. 12). The above definitions stress the mechanics such as the rules, or objectives such as strategy, chances of winning (luck) and the players’ engagement with the game, but not with each other.

Other definitions mention the type of interaction among players. Cruickshank & Telfer’s (1980) definition describes the interaction as a competition : "*competitive interactions* bound by rules to achieve specified goals that depend on skill and often involve chance and an imaginary setting" (p. 262). They also integrated personal aspects such as skills and imagination.

In the present study, social interaction among players and their participation during the game are viewed as important aspects of games. In every game, there is a certain level of interaction with both oneself and other players. In addition, the game’s environment should be a safe social place where players can make mistakes and explore strategies (Hays, 2005).

Some researchers have considered game elements and the social aspects when defining games. For example Gough’s (1999) definition: “a game needs to have two or more players, who take turns, each competing to achieve a ‘winning’ situation of some kind, each able to exercise some choice about how to move at any time through the playing" (p. 23) and Boller’s & Kapp (2017)stated that : “A game is an activity that has a **goal**, a **challenge** (or challenges), and **rules** that guide achievement of the goal; ***interactivity*** *with either other players* or the **game environment** (or both); and **feedback** mechanics that give clear cues as to how well or poorly you are performing. It results in a **quantifiable outcome** (you win or lose, you hit the target and so on) that usually generates an **emotional reaction** in players” (p. 4) (the terms bolded by the researchers are e detailed in section 3.4). These definitions refer to the players’ interactions as well as the players' ability to make decisions by choosing their moves according to the rules. Affect however is beyond the scope of this research. Game elements play an important role in their design. In addition, when evaluating a game, its elements can be identified and assessed. The evaluation of educational games as well as their designs should examine how these elements contribute to the alignment between the game’s objectives and the learning objectives (Kalmpourtzis & Romero, 2020) as detailed in the following section.

3.4 Evaluation of games and their design.

When designing a game, different game elements need to be taken into consideration. For instance , Boller's and Kapp's (2017) basic elements that are mentioned in their game definition (in chapter 3.3) refer to:

- Having a specific **goal** (which is the differences between a game and play) which dictates what players need to do to win. When referring to how players win the game the terms core dynamics or game objective are used.
- **Challenge** in the game should be in the zone of proximal development of the players to prevent them from giving up.
- **Rules** are the element that gives a structure to the game.
- **Interactivity** refers to the communication among players and game rules.
- **Game environment** is defined as the special space created as students play and contains the social norms, its own rules and challenges.
- **Feedback** mechanisms not only cover direct feedback provided by the computer but the entire way of comparing the players' game situation to the goal (how far they are from winning) and other players in the game.
- **Quantifiable outcomes** indicates when the game is over (or a level was completed) as agreed upon by all the participants.
- **Emotional reaction** is usually triggered by players' moves and strategy throughout the game.

These game elements and components are referred as game mechanics. The actions carried out by players to winare referred to as the game objectives (Kalmpourtzis & Romero, 2020).

When designing educational games researchers refer to a broader perspective that connects game mechanics and game objectives with pedagogical goals. Aleven, Mayers, Easterday & Ogan (2010) presented a framework that consists of three components that caninteract with each other. The first component is learning objectives. This refers to the way designers incorporate a coherent set of educational goals into the game. This component includes which prior knowledge and skills players should have before playing, which pedagogical knowledge they need to retain as the play the game and the potential for transfer after the game.

The second component of mechanics, dynamics and esthetics (MDA) appear as three interrelated levels. The game mechanics, which are elements consist of such as materials, rules, explicit goals, basic moves, and control options, etc. available to the players. Dynamics refers to players' behavior, which is usually influenced by the game mechanics. Aesthetics refers to the players' emotional response.

The last component is instructional principles; for example, giving information on-demand and the just-in-time principle. This principle suggests providing explicit information when the player needs it to make a needed move in the game. Components in the framework support and complete each other to create a more coherent connection between the game and learning objectives. Therefore, educational games should be designed with these game elements and learning objectives in mind.

Kalmpourtzis & Romero (2020) suggested a similar notion of coherence as a way to assess educational games. Based on the Constructivist approach, they applied the Constructive Alignment (CA) framework. Constructivism refers to the way the activity or game enable students to build their own knowledge. Alignment refers to the way educators aim to achieve a learning outcome. They focus on the alignment of two sets of components: learning and game mechanics as well as learning and game objectives. The learning mechanics has to do with the learning facet of the game such as asking questions and exploring. When designing and evaluating a game, its mechanics should be aligned with the learning mechanics to avoid cognitive load. Another important aspect in assessing educational games has to do with the level of coherence between learning and game objectives. The learning are objectives composed of the skills and knowledge students are expected to learn and retain after they play. Learning objectives should be achieved through the objectives of educational games (the actions players need to take in order to win the game). This is an iterative designing-evaluation process that is based on game elements and the coherence between learning objectives, game objectives, learning mechanics and game mechanics.

Other educational game evaluations can be carried out by questionnaire to measure students' experiences after playing (Petri, Wangenheim & Borgatto, 2016). For instance, the Model of the Evaluation of Educational Games (MEEGA+ model) is a questionnaire that examines the three dimensions of motivation, user experience, and learning. Interviews of players are a common way to evaluate game design (Amory & Seagram, 2003).

Evaluation can also take place through comparative research. For instance, Jantan's & Aljunid (2012) evaluated a game design by comparing the learning outcomes of two groups of students who played two different versions of agame .

Since this research is strongly based on students' modes of participation in mathematical discourse while playing in groups, a socio-cultural approach relating to both discourse and mathematics was needed. This perspective can shed light on game design and evaluation in a different way than assessing mechanics or learning objectives. The Commognitive framework (section 3.5) is rooted in the socio-cultural approach and provides operational tools to examine students' participation in mathematics discourse.

3.5 The commognitive framework

Commognition is a socio-cultural theory for the study of learning that views communication and discourse as central to human thinking and learning (Sfard, 2008). According to Commognition, mathematics, similar to any other discipline, is seen as a special type of discourse with unique ways of saying and doing. Learning mathematics is considered the process by which students change (improve) their communication about mathematics by adopting historically established mathematical discourse. The word *commognition* is a combination of communication and cognition, because thinking is considered to be communication with oneself.

3.5.1. Characteristics of discourse

Discourse is defined in the commognitive framework as a form of communication characterized by special words and their use, visual mediators, endorsed narratives and routines (Sfard, 2008). The following characteristics apply specifically to mathematics discourse. *Words and their use* include using specific vocabulary in specific ways, such as "minimum vertex", "angle", "function" and so on. *Visual mediators* are tangible entities that facilitate communication about mathematical objects and ideas. In games for example, some visual mediators relate to mathematical objects, such as a geometric shape, while others are specific to the game, such as the path of ellipses on a boardgame. *Endorsed narratives* are utterances that describe mathematical objects and connections between objects or processes which may be endorsed. These narratives include mathematical statements such as axioms or definitions accepted in the mathematical community. For example, a mathematical narrative that relates to the quadratic function is that its graph always intersects the Y axis once.

The fourth characteristics is *routines. Routines* are patterns that are enacted as a reaction to a situation which the user considers to be similar to others. An example of a routine performed in class is solving a quadratic equation by using a given formula; e.g., solving $0 = x^2 + 2x - 15$ by applying the quadratic formula $x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ or by applying the factor routine $0 = (x + 5)(x - 3)$. A different type of routine performed in class is raising one's hand to signal a request to participate. Lavie, Steiner and Sfard (2019) conceptualized routines into two components: the task that the performer of the routine aims to achieve, and the procedure to achieve the task. Thus, the task of the routine above is to solve a quadratic equation, and the procedure includes using the quadratic formula or the use of factoring.

Sfard and Lavie (2005) distinguished between three types of routines: deeds, rituals and explorations. This study only focuses on the two discursive routines of rituals and explorations. The performer of a ritual routine focuses on actions rather than on the result. This includes rigidly performing previously learned procedures, often for social recognition, and to interact with an adult or an expert. Students who carry out ritual routines often depend on an external authority. Thus, ritual participation, which is participation that involves mainly performing ritual routines, is process-oriented (Lavie, Steiner & Sfard, 2019).

On the other hand, performing an exploration routine refers to the process of producing mathematical narratives for their own sake (Heyd-Metzuyanim, Tabach & Nachlieli, 2016; Sfard & Lavie, 2005). Performing exploration routines, or participating exploratively, is characterized by focusing on the goal, exercising agentivity and showing flexibility regarding the procedure performed (Lavie, Steiner & Sfard, 2019).

3.5.2 Learning as routinization and the process of de-ritualization

Learning involves becoming more proficient in performing previously established routines. That is, learning can be conceptualized as a process of routinization (Lavie, Steiner and Sfard, 2019). Studies show that learning usually initiates in more ritual performance of routines (Lavie, Steiner & Sfard, 2019; Nachlieli & Tabach, 2012). As students engage in relevant opportunities to learn, their communication about the subject gradually changes in that they become more independent, make mathematical choices and can justify them. Their participation becomes more explorative (Sfard & Lavie, 2005). This process is called de-ritualization and refers to the development of students' participation from being more ritualistic to more explorative (Lavie, Steiner & Sfard, 2019). Lavie, Steiner & Sfard (2019) placed the characteristics of participation and routine performance on a ritual-exploration continuum. Thus, instead of characterizing learners' participation as more ritualistic or explorative in its entirety, different aspects of participation are considered separately. These include flexibility, substantiability, agentivity, bondedness and applicability. However, the present study only looked at students' participation in group games. In other words, changes in individual students' discourse were not assessed. Rather, the analysis deals with the ways in which students in a group participate in mathematical discourse while playing.

Flexible participation can involve either suggesting more than one procedure to solve a task, or by using a certain procedure to solve different tasks (Sfard & Lavie, 2005). A solution or explanation is considered *bonded* if the output of each procedure/step is the input to the next step until the student finds the answer. *Applicability* refers to the range of the students' precedents from previous procedures and mathematical narratives that students use to solve a task. When encountering a mathematical problem, students apply previously learned routines. Performers' *agentivity* refers to students' independence when deciding what task and which procedure to choose. *Substantiability* relates to the justification of mathematical narratives and the procedure that leads to them (Lavie, Steiner & Sfard 2019).

3.6 Research questions

a. What are the characteristics of students' explorative participation while playing games in middle school mathematics classrooms?

b. Which characteristics of game design promote or hinder explorative participation?

4. Method

In this study, I was the researcher, the teacher and the game developer. I taught the class that participated in this study for three years, and developed the games and used them in other classes for several years prior to this study.

4.1 Data collection

This research was conducted in a middle school in the center of Israel.

As a teacher, I find it important to investigate teaching and ways to improve it. Therefore, I occasionally videotape my lessons, and in particular lessons in which students play games. The data for this study consisted of some of these videotapes. Using the videotapes of students in their natural learning environment at school provides an authentic opportunity to examine students' natural forms of participation in mathematical games.

Overall, I randomly videotaped six lessons in 8^{th} and 9^{th} grades during lessons in which mathematical games were played. The students were used to playing mathematics games as an inseparable part of their mathematics learning since 7^{th} grade. This is important since learning how to learn while playing, and accepting games as a part of the mathematics classroom is a process that often takes time. In this study, I focus on students' participation beyond those initial stages.

In the six lessons, thirteen groups of students were videotaped playing six different games. The videos were taken using cameras fixed on a tripod, that were usually oriented towards the table and game, not the students' faces.

Each group was videotaped for about 35 minutes. The videotaping took place during the last semester of school in 2017/2018. For the present study, five out of the thirteen videotapes were fully transcribed and analyzed. The videotapes were chosen retrospectively after receiving permission from the Chief Scientist at the Ministry of Education and the Ethics Committee of the Technion.

There were four main reasons for selecting these videos, in addition to centering on students in 9^{th} grade who are used to playing math games in class. First, I wanted data documenting different levels of achievement within the same grade level. Therefore, the videotapes of the ninth grade were chosen over those of the eighth grade, since they included an advanced class and a Telem (underachiever) class (detailed in section 3.2). Second, I wanted to analyze students' explorative participation in different types of games so that the findings would not be limited to one specific type of game. I therefore chose one computer game ("Catch the Stars") and two games that did not use technology. Third, I wanted to explore different mathematical contents. Therefore, I chose one game about geometry (the "Totem" game, which was the only game in geometry). The algebra game ("Like Terms") was chosen since it was the only game played with the Telem class. Though these games differ from each other in mathematical content and type, they correspond to the same game definition. Based on the definitions for games reviewed in section 3.3, the games used in this study could be characterized as following: "a competitive interaction based on mathematical principles. This interaction must include at least two players that are committed to the same set of rules, and are free to choose their own strategy and moves, based on mathematics and luck".

All the videotaped games are summarized in Table 1. The names and groups marked by a * were selected as data for this study, and therefore fully transcribed and analyzed.

Name of the game	Type of game	Content	Participants
***Totem**	Board game	Geometry - the quadrilateral family	Three groups from the Advanced class, 9th grade (only videotapes of **two groups** were used for this study). Four players played in each group.
***Catch the Stars**	Computer game	Functions - quadratic and linear functions	**Two groups** from the Advanced class, 9th grade
***Like terms**	Blocks game	Combining like terms	**One group** from a Telem class, 9th grade. Two students played in the group.
River crossing game	Board game	Probability	Three groups from the Advanced class , 9th grade
Lucky wheel	Board game	Probability	One group of 8th graders and one group from the advanced class in 9th grade
Find the algebraic way	Card game	Simplifying algebraic fractions	Two groups from the Advanced class , 9th grade.

*Table 1- Games played in the videotaped lessons. * lessons analyzed for this study.*

4.2 participants

The sample was made up of 14 ninth grade mathematics students from two classes: an advanced class (12 students) and Telem class (2 students), as detailed below. The 14 students were part of five groups (Table 1).

The advanced class

This class is the highest (A) of three levels (A, B and C) in the 9th grade of the school. Students in this class are encouraged to take 4 and 5 units of math in high school. Most of the students in this group were enrolled in my class in 7th and 8th grades. Therefore, they were used to playing games during mathematics class. They were also used to playing in groups and to being videotaped for pedagogical purposes. In these classes, groups formed spontaneously and changed from one game to another. Overall, the data from four groups from this class (including 12 students out of 36) were analyzed in this study.

Telem class

The Telem class was composed of 30 low achievers (at least 4 failing scores in 8th grade). Most of them had emotional problems, behaviorak problems, and/or cognitive difficulties. The class was divided into two groups during the mathematics lessons. I taught the group with the 20 most challenging students. I also integrated one student from a special education class who expressed high motivation to learn mathematics and behaved well. Two students from this class appear in the analysis, one of whom is the integrated student from special education.

4.3 Mathematical games in the study

In this section, the rules of the mathematical games referred to in this study are described. The three games chosen are characterized as detailed in Section 4.1. Each game requires more than one player and encourages students to make moves based on mathematical decisions.

The "Totem" game

Totem is an educational game for a group of two to four players that I invented inspired by the rules of the Totem board game marketed by the Tactic Company. This game focuses on the geometric properties of the quadrilateral family and encourages students to discuss geometric objects and theorems.

Game pieces: The game consists of a board of 10X10 ellipses (Figure 1). The names of quadrilaterals (rectangle, square, trapezoid, isosceles trapezoid, rhombus, kite, and parallelogram) are printed on 96 ellipses and four ellipses are empty. Each of the four players have four plastic chips of the same color. A common pile of 25 property cards is placed (face down) next to the board game. On each card, a geometric property is written, such as “all sides are equal”.

Figure 1- Totem board game, plastic chips (yellow and red) and the pile of property cards

Game goal: The object of the game is to move all four plastic chips from the start position (the four ellipses in the center of one side of the board) to the other side (to the four ellipses on the other side of the board). For instance, in Figure 1 the red player needs to position her four red plastic chips on the ellipses occupied by the yellow's plastic chips to win the game.

Mathematical moves: Laying a plastic chip on an ellipse is possible only if the quadrilateral printed on it matches the current property card in the round. For example, if the property card says, "all sides are equal" players can only progress on ellipses that say rhombus or square until the next card is turned over.

Game rules: Players move in turns across the board according to the property card presented face up to all players. The group plays according to the same property card until a plastic chip is positioned on an empty ellipse. Each player is allowed to go over as many ellipses as she wants in the same turn, as long as all shapes in her path match the given property card. Players can only progress on the board in a straight line (horizontally or vertically but not diagonally). Directions can be changed from one turn to another but not in the same turn. On each turn, the players must make a new decision about which direction they want to take to proceed in the game.

When two players reach the same ellipse, the last player is allowed to move the existing player in any direction she wants according to the game rules.

The "Catch the Stars" game

This game focuses on creating parabolic and linear graphs by writing their algebraic expressions and setting their boundaries (Figure 2).

Figure 2- the upper screen shows a launched ball after the screen was solved by adding the red parabola. The lower screen shows that the screen was successfully completed.

Games pieces: This game is part of a computer application called Desmos[1] where students can create and/or play with graphs on the computer. This game was the end-product of a learning unit on quadratic functions and was co-created by the students and myself.

The students created a screen (which was later part of the game) according to my instructions. The game consists of 36 screens (as shown in Figure 2). Each screen has ten yellow stars, a number of graphs and a launch button on the right side of the screen. The launch button releases a purple ball (Figure 2). Once the ball goes through all ten stars, the word "success" appears on the screen in green and that challenge is completed. On the left side of the screen, there is a table with the algebraic expressions of functions. The expression and its graph are in the same color.

Game goal: To catch all the stars on the screen with the ball. A star is said to be caught if the ball passes through it. The winner is the group that solves the highest number of screens by the end of the lesson. The teams' positions are projected on the board. The ball roles on graphs. A graph can only be created by typing its equation into the table. Creating only part of the graph can be done by typing its x or y range in brackets ({ }) (defining its boundaries).

[1] Desmos (www.desmos.com) is an advanced graphing calculator that can be used on the computer or mobile. In addition, a section of templates to self-create games such as *Catch the Stars* is included.

Mathematical moves: Groups are expected to graph parabolic and linear shapes by writing the appropriate algebraic expression and setting its boundaries, if needed. The computer simultaneously shows the graph of the typed algebraic expression (see Figure 2). Players are also allowed to set the boundaries of the graph by adding the range of X's. Players have to decide which graph they want to make according to the star and ball position. They can choose any form of expression (vertex, standard or factored form).

Games rules: Since the winner has to solve as many screens as possible in any order, groups are allowed to skip screens. Groups are not allowed to change the existing graphs on the screen (these are referred to as the setting or background) though it is technically possible. They can only add new graphs. There is no limitation to the number of added graphs. The screen is only considered solved when the word 'success' appears on the screen.

The "Like terms" game

"Like Terms" is a (non-digital) educational game that focuses on equivalent expressions for a standard form of quadratic functions.

Game pieces: A box with 50 tiles (Figure 3b). Three types of variables appear on the tiles: x^2 terms, x terms and numbers. The algebraic coefficients of the terms as well as the numbers range from -6 to 6. On the other side of each tile there is a negating term. For

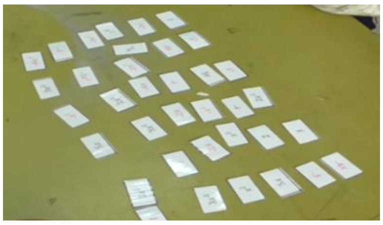

Figure 3a - starting position (6x6) tiles

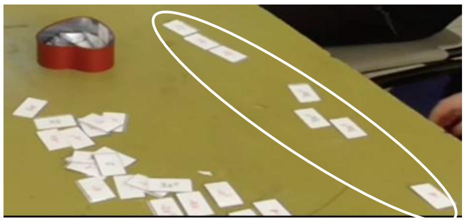

Figure 3b: final expression (in white ellipse) and the tiles' box

example, (-5x) and 5x are written on the two faces of the same tile. Students can play in groups (of two) or individually against the rest of the class.

Game goal: To create an equivalent algebraic expression to the one written on the board (by the teacher), as fast as possible (the manual leaderboard is updated by the teacher at the end of every round). The teacher keeps track using a manual leaderboard on the whiteboard. Students can compete against each other (playing in pairs) or against the rest of the class.

Mathematical moves: The terms written on the tiles are always lower than the terms on the board. For instance, when trying to make the expression $10x^2 + 9x + 12$, tiles such as $10x^2, 9x, 12$ do not appear on the board. To create an equivalent expression, terms on the tiles must be combined by adding a term or number to another, such as 4x+5x creating 9x. The player has to combine at least two terms together. A player is allowed to use either side of the tile (positive or negative). The only arithmetic operation allowed between terms and numbers is addition.

Game rules: Each player randomly picks 36 tiles from the box and arranges them into a 6x6 square (Figure 3a). Players can only use terms or numbers from the 36 tiles. At the end of each turn, new tiles from the box replace the used tiles (such as those in the ellipse in Figure 3b). Students can flip a tile if needed.

4.4 Analyzing the data - first RQ

The first RQ was: *What are the characteristics of students' explorative participation, while playing games in middle school mathematics classrooms?* To address this question, the data were analyzed as follows. First, excerpts of the mathematical discourse to be analyzed were identified. Then, these excerpts were analyzed according to five characteristics of participation, using a methodological tool that I developed for this study. Each step is detailed in the next sub-sections.

4.4.1 First step in the analysis: Identifying mathematical discourse and game discourse

To address the first RQ, all excerpts corresponding to the students' mathematical discourse had to be distinguished from other discourse that took place while playing. In this process, it became apparent that students talked and performed actions either about mathematics or about the game, or about both. They hardly talked about anything else. Therefore, a choice was made to identify turns as belonging to each of these discourses (mathematical or game), according to four discourse characteristics: words, visual mediators, routines and endorsed narratives (Sfard, 2008), as detailed in Table 2. To clarify how this was done and explain the rationale considerations and choices taken, three examples are presented.

Characteristics	Operationalization	Examples

Word use	Is the lexical choice about the game-play or mathematics (or both)?	Lexical choices for game-play: tiles, turn, strategy, game rules. Lexical choices for mathematics: quadratic function, square, terms
Visual mediators	Does the group refer to the visual mediators of a game or mathematical objects?	Game-play: The chip reflects the player's actions on the board. Mathematics: The name of the quadrilateral reflects their geometric shape.
Routines	Does the group perform game routines or mathematical routines?	Game routine: developing or applying a game strategy Mathematical routines: using deductive logic; writing boundaries for the graph of a function; Creating an equivalent expression.
Endorsed narratives	Does the group participate according to rules that regulate the game-play or the mathematics discourse? Are the uttered or referred to narratives about the game or mathematics?	Game play: playing in a certain turn sequence. Mathematics: combining the same type of variables. Using the definitions of quadrilaterals.

Table 2- Characterizing mathematical discourse and game discourse

The first example comes from an excerpt that was identified as part of mathematical discourse and appears in yellow (Excerpt 1 below). Use of mathematical words such as the number word "three" (5, 6), routines such as "move it down" (7) can be seen. In this context they mean "change the number of the parameter to change the parabola so that the graph will be placed lower on the Cartesian plane" and the use of graphs as mediators of function throughout the excerpt. The students looked at the screen throughout the whole game. On the screen they could see how the graphs were being generated by the computer as they typed their algebraic expression. The discourse characteristics had to be within a mathematical context. For instance, a sentence such as "you have to move all of your four plastic chips" was not considered a mathematical discourse merely because it contained the word "four". This is because 'four' in this context was related to a game element (the four plastic chips that a player uses to execute moves in the game). The word "move" in this sentence is not in the same context as the word "move" in turn 7 (Excerpt 1).

Excerpt 1 – Mathematical discourse while playing Catch the Stars

#	Name	What was said [what was done] (more information)
5	Ohad	That's it, lower it (the number of parameters in the standard form of the quadratic function). ah… three three (means 3 in 0.3)
6	Shalev	What? It's three? [typing the number in the algebraic expression]
7	Ohad	Yes, no ah… two (means 0.2) and move it (the whole parabolic graph) down

Excerpt 2 is an example of game discourse. Students are discussing the game rules since they disagree how to apply them. The excerpt evidences the use of words such as "turn" (6, 9) while referring to a turn in the game, and "move" (8), referring to moving a chip on the board. The visual game mediators were the plastic chips that represented each player. Game routines can involve moving the chips on each turn, and endorsed narratives were considered to indicate how to execute the move on each turn according to the point in the game. This excerpt was identified as part of game discourse and therefore colored green.

Excerpt 2 – game discourse while playing Totem

#	Name	What was said [what was done] (more information)
6	Tehila	But Sasha, now it's Anna's turn
7	Sasha	[moving his hand counterclockwise] (hand gesture is an answer/explanation for executing his turn)
8	Anna	No, everyone can move I think (meaning that there are no turns, but everyone can move on the board at the same time)
9	Tehila	No, it's everyone in his turn (authoritative and impatient intonation) (means that every player can only play in turn)

Identifying the type of discourse of an excerpt was not always straightforward. In some ambiguous cases, the context of students' discourse and their goal was further analyzed to identify the type of discourse. In Excerpt 3 for instance, Raffi claimed that he did not know the answer. His words were first classified as mathematical since it was an answer to the mathematical task of *which quadrilaterals match the property of "two pairs of opposite equal sides"?* However, Raffi's intonation of arrogan and the way he stressed the fact that he did not know the answer were strange and unusual. In addition, Anna's reaction to Raffi's words was surprising "what a shit you are!" (172). Why would Anna suddenly humiliate a student who claims not to know the answer? In this case, although the discourse characteristics matched the mathematical discourse, looking at the broader context revealed a different picture. In turn 143 it emerged that Raffi suggested adopting a game strategy where players should not be considerate to each other. Raffi seemed to mean that they should play using a strategy of ‘every man for himself’ and not as a group that helps each other find the solution. In turns 165 and 169 he executes this game strategy by choosing not to contribute to the mathematical discussions in the group that Tehila tried to initiate (164). Anna found this position unacceptable; hence her comment (172). Therefore, turns 165 and 169 were considered here as part of the game discourse (colored green).

Excerpt 3 – ambiguous discourse while playing Totem

#	Name	What was said [what was done] (more information)
143	Raffi	From now on, let's be less nice
….		
164	Tehila	wait a sec, let's look for, it could be also a rectangle wait also a rectangle and also a parallelogram
165	Raffi	I have no idea (arrogant intonation)
166	Sasha	Rectangle (approving Tehila’s answer)
167	Anna	Also a kite?
168	Sasha	Kite … (considering) no. A rectangle
169	Raffi	I don’t know
…		
172	Anna	What a shit you are!
173	Raffi	Just kidding

4.4.2 Second step in the analysis: defining the unit of analysis and adjusting the methodological tool

In this section, the steps taken to analyze the excerpts are detailed. The data were segmented into episodes (what Van Dijk, 1981, refers to as semantic units or episodes). This is described in the first sub-section. Then the methodological data analysis tool is presented along with considerations for using this tool.

The analysis, which focused on students' participation, was performed separately for each task in the game. Each task was defined as a new episode which constituted a round (Table

3). Episodes are often characterized by a change in the main mathematical goal (ibid, 1981). In the games chosen for this study, each round of the game focused on a different mathematical task. Therefore, a round in each game was considered an episode and was analyzed separately. The description of *round* in each game is shown in Table 3. During each round, students took turns to make their moves. Students in the groups (as presented in Table 3) discussed each new mathematical task.

Game	Round	Mathematical task presented by the game
Totem	From the time the group reads a new property card until a player steps on an empty ellipse and a new card is picked.	In every round, a new property card is presented, and the task is to produce narratives that connect a given property and a quadrilateral.
Catch the Stars	From the time the group enters a new screen until they press the arrow to move to the next screen.	In every round, new graphs and stars are presented. The task was to create algebraic expressions and set their boundaries to graph linear and parabolic graphs to collect all the stars.
Like terms	From the time the teacher writes the parabolic expression on the board until she erased it	In every round, a new algebraic expression in the form of $ax^2 + bx + c$ is presented. The task is to create an equivalent expression by using the given terms that appear on the tiles.

Table 3- Descriptions of the rounds in each game

4.4.3 Third step in the analysis - the commognitive methodological tool

The analysis of discourse in this study was conducted at the group level. Sfard and McClain (2003) described the ways in which a group does mathematics as a form of joint meaning-making involving collective effort. While playing, the members of the group discuss mathematical objects, routines, and narratives together as part of the game. The goal of the analysis here was to examine students' participation in groups while playing mathematical games, through the lens of ritual-explorative participation.

The operationalization of each of the characteristics of participation was based on definitions in the literature and the characteristics of de-ritualization (Lavie & Sfard, 2005; Lavie, Steiner and Sfard, 2019). The tool considers the adjustments that make it possible to apply these characteristics to the game context. Three specifications of the context were taken into consideration: the discussions took place in groups of at least two students, the context was game-playing during mathematical lessons, and lasted for periods of 30 minutes rather than being longitudinal. Each specification is described below in relation to the specific characteristic of participation.

One methodological choice was that group performance was defined as including all the actions and suggestions performed by any group member during one round of the game. The goal was to refer to what was said and done mathematically among the players in the group and not specifically by one student to another. The aim is to try to capture the ways in which students have the opportunity to participate in mathematical discourse while playing. Considering the *round* as an episode related to the context of game-playing. While playing, I expected that students would talk about the game, the mathematics involved in the game, and perhaps other topics. Another methodological choice was that only those parts of the game-play in which the students talked about or did mathematics were analyzed. However, game discourse was considered when it could shed light on the mathematical discourse.

This study was deliberately not longitudinal. That is, the students' changes in performance over long periods of time were not examined. Rather the data constitute "snapshots" of students' performance and the analysis aimed at characterizing students' performance at a given time. These methodological choices are discussed next.

Agentivity

Lavie, Steiner and Sfard (2019) defined the exercise of a performer's agentivity by stating that "the performer does not need another person's invitation to engage in the performance of the routine: She is now capable of setting the relevant task situations for herself, in response to her own needs." (p.170).

In this study, a group was considered as exercising agentivity when it independently chose relevant tasks and procedures in the process of solving the mathematical problem presented in the game. This allowed for cases in which some of the group members were less engaged in choosing tasks and procedure or made a minor contribution to the process of solving the problem. In these cases, if the group kept choosing tasks and procedures, it was still considered as agentivity. A lack of agentivity was said to occur when the group stopped solving the mathematical problem or when they asked someone outside the group to help them.

The question which determined the agentivity of the group was: *Does the group set relevant tasks and independently choose relevant procedures to solve the tasks?*

In the games referred to in this study, some game context gave students opportunities to give up trying to solve the mathematical problem. In the Totem game and Like Terms game, students are allowed to skip a turn. In Catch the Stars students can skip a screen and renounce solveing the mathematical problem. Though a possible move in the game, skipping a turn was considered as lack of agentivity, since the analysis focused on the mathematics discourse rather than game discourse. The choice to skip a turn was considered part of the game discourse, in that it is a strategy to improve the players' situation in the game. That is, in terms of agentivity, the game does provide players opportunities to conceder their agentivity. It is a question whether players opt for this choice or continue to struggle to solve the mathematical task.

The analysis of agentivity is exemplified in Excerpt 4. One of the students in the group did not set any tasks or suggest any procedures. He even said explicitly that he could not take part in this round. The other student was very independent and insisted on suggesting mathematical procedures.

Excerpt 4 – individual agentivity versus group agentivity

#	Name	What was said (meaning) [what was done]	Agentivity
25	Ron	I don't have I don't have anything that relates to eight ($8x^2$) or nine ($9x$) and not to seven (7)	Though Ron claims he cannot continue with the game, it appears that he based his reaction on a mathematical decision. Therefore, I considered it as exercising agentivity. His task was to match the exact terms as in a memory game. He was looking for a tile with the term $8x^2$ on it.
26	Dan	Do you need me to help you? [moves his chair closer to Dan's desk]	Dan offers help solving the task, which is considered as exercising agentivity.
27	Ron	I don't need you to help me, I have none (of the given terms)	Lack of agentivity is seen here since Ron is not willing to get into a mathematical discussion about his claim that the mathematical problem cannot be solved.
28	Dan	What do you have? (on the tiles)	Dan insists on looking for a solution by concentrating on what is given on the tiles.
29	Ron	I don't have I don't have (nervously)	Ron refuses to get into a discussion about possible solutions.
30	Dan	Do (pick) eight ($8x^2$) turn it [takes a tile with -3x, and flips it and separates it] this three (3x), plus nine and here [turns to Ron] do six	By insisting on finding a procedure to the task of creating an equivalent expression to

		($6x^2$) [picks $6x^2$ and separates it] find me another 2 ($2x^2$) like this (meaning x^2). Here two [picks a $2x^2$ tile and places it near the $6x^2$] you already have one (of the terms in the target expression) you see you already have one. That's how it has to be done	$8x^2 + 9x + 7$, Dan maintains the group agentivity.
31	Ron	What? (surprised) I understand. Here, let's take two, we'll take two (funny unclear voice) [picking a card of x squared and putting it back. Later on he picks 3x and 5x]	Dan has captured Ron's attention and engaged him with a possible solution. Later on, they produce an equivalent expression together.

Since this round eventually included setting the group's task (finding an equivalent expression for $8x^2 + 9x + 7$) and group's procedure (adding terms on the tiles until a new equivalent expression is found) it was considered to display the group's agentivity.

Applicability

Lavie, Steiner and Sfard (2019) described applicability as "considering the range of task situations for which its performances so far are likely to constitute precedents." (p.169). In other words, applicability is the students' use of precedents that they found to be relevant for a certain task. In this study, applicability was operationalized by the following question: *Is the group's routine performance based on mathematical precedents which were produced during the game (by the game design or by another player), or based on mathematical precedents which were produced before the game?*

This question addressed both the applicability based on mathematical narratives and routines learned before the game and during the game.

The Totem game related to the properties of quadrilaterals, most of which had been studied in class before students played the game. Catch the Stars was about the connection between graphs (linear and parabolic) and their algebraic expressions, which had also been learned previously. These two games were played in the advanced class. Thus, I expected to see how students apply former narratives and routines. It was not clear which precedents if any students in the Telem (low achiever) class would apply.

In the initial stages of the analysis process only phrases that referred to precedents such as "I remember that we learned about…" were considered as indicating applicability. These declarative sentences pointed to students' application of former procedures and narratives. However, there were additional pieces of evidence for applicability.

The first type of excerpts included group discussions in which a student stated that she did not have a procedure to solve the problem but another student suggested one. The student then applied it to solve the task. The second type of excerpts was composed of students' use of specific procedures that were assumed to have been applied in the past. The assumption was that students who applied a specific procedure (such as vertex representation of a quadratic function) were probably familiar with the procedure. As a result, the group applicability tool assessed both explorative participation of procedures students became familiar with during game-playing as well as previous applied procedures.

Bondedness

Lavie, Steiner and Sfard (2019) described bondedness as follows: "if the output of any given step in its procedure, if not yet the desired final product, feeds in (is used as an input) latter steps." (p.168). They illustrate this by a process where a young child learns that the procedure of counting blocks, which eventually leads him to answer the question of how many blocks there are. In other words, the enumeration ends up as the answer to the question.

In the current study, bondedness was considered to have been manifested by one student's bonded steps in a procedure or by the whole group; i.e., when the output of a step suggested by one member of the group became the input of another member's step. Bondedness was thus operationalized by the questions of *Does the outcome of each mathematical step in the procedure feed into the next one?* And *Are all the steps in the procedure relevant to achieving the desired task, or are some redundant?*

Although bondedness is considered low when the output of one step is not the input of the following step or when steps are redundant, here it was not considered low when it was part of group discussions. This is because the groups often elaborated the solution together by trial and error until a full solution was found. Excerpt 5 is an example of a group bonded routine. In this excerpt, the students elaborated the narrative that the quadrilateral matches the property card together, and gradually reached a conclusion.

In Excerpt 5, the students discussed the property card of "every pair of adjacent angles sum to 180 degrees" and tried to identify all the quadrilaterals on the board that fulfilled that property. The students were not sure about some of the suggested answers such as the trapezoid and rhombus, and through discussion, they developed their narrative of the solution together.

Excerpt 5 – group bondedness in the Totem game

Turn	Name	What was said	Output (which became the input of the following step)
540	Galia	Every pair of adjacent angles equals 180 (the property card). In a trapezoid (hesitation)?	Input - Reading the card Output- suggesting quadrilaterals that meet the property
541	Anna	And parallelograms	Output - Considering others' suggestions
542	Raffi	Yes, she is right	
543	Anna	Trapezoid (hesitation)	Output - Considering others' suggestions
544	Sasha	Rectangle, square (confident) ahh trapezoid (hesitation)	Output- Suggesting quadrilaterals such as rectangles and squares Relating to others' suggestions Like trapezoids
545	Anna	Parallelogram (hesitating tone)	
546	Sasha	Para(llelogram) parallelogram	
547	Raffi	Everything (quadrilateral) that has 90 degrees (angles) or that is a parallelogram.	The output of the suggested quadrilaterals (543 - 546) led to a new input – Raffi mapped the answer into two alternatives : (1) quadrilaterals with angles of 90 degrees (2) parallelograms
548	Galia	So it's not a trapezoid?!	The generalizing input results in a new output: the exclusion of the trapezoid because it is not a parallelogram, and also because it does not have equal angles.
565-595		(A short class discussion. The conclusion expressed by the teacher is that two pairs of parallel sides are required to match the property card.)	The input of the class discussion is that every quadrilateral that has two pairs of parallel sides matches the property card.
619	Sasha	A rhombus, parallelogram, square and rectangle (matches the property).	The output of this discussion is summarized by Sasha without sorting

…		(Rhombus is questioned in the group)	
638	Raffi	What? who said that in a rhombus the (all) the adjacent angles (sum to) 180?	checking the output of turn 619
639	Galia	A rhombus is a type of parallelogram	Making the implicit input and output (548) explicit Input- a generalization of input (2) in turn 547. Types of parallelograms such as a rhombus also match the property card Output- a rhombus matches the card

The bonded procedure in this episode consisted of the following:

a. Read the property card (541)
b. Consider specific quadrilaterals that exhibit the given property and those that do not (541-546)
c. Produce a narrative that generalizes which quadrilaterals have the given property (639)
d. Examine whether the suggested quadrilaterals fulfill the suggested narrative (548, 619, 638-639)

Substantiability

Lavie, Steiner and Sfard (2019) described the substantiability of routines as the learners' reasoning or justification of their performed routine. In the current study, the students' justifications of their own solution or the solution of others in the group to justify a certain outcome was considered evidence of substantiability.

Substantiatiability was identified by the operational question of *Does the group justify the suggested solutions?*

Only mathematical justifications were considered evidence of substantiability .

Flexibility

Lavie, Steiner and Sfard (2019) described flexible performance as suggesting more than one procedure to solve a given task. They say that: "This happens when the child realizes that other, hitherto unrelated, procedures can be used to perform the same task" (p. 167). Here, again, they exemplified flexible performance of routines with the case of Milo, a toddler who realized that to discover which pile has more blocks, he could either compare the piles visually or align them as towers.

When focusing on flexibility in a group, I considered the performance and suggestions of all the participants in the group. That is, if more than one procedure was suggested to solve the task by one or more of the participants in the group, I defined the performance as flexible. Differences in the suggested procedures differed across games. Flexibility was thus operationalized by the question: *Does the group apply different procedures to the same mathematical task?* Descriptions of what was considered "different procedures" in the different games are described below.

In the Totem game, the players' main task is to produce narratives about properties of quadrilaterals (e.g., in a parallelogram, there are two pairs of opposite equal sides). The procedure used to solve the task could be a deductive proof presenting a particular drawing or numerical example, or retrieval of familiar narratives. However, the procedure did not need to be explicit. That is, players were not required to reason how they produced the narrative or substantiate it. A performance was considered flexible if more than one procedure to resolve the task was used by the participants in the group, during one round of the game.

In Catch the Stars, flexible performance was evidenced by suggesting different types of graphs (linear or parabolic) or combinations of graphs, and as a result, suggesting various appropriate algebraic expressions. Flexible performance was also considered to be evidenced in different choices of representation of the graph (vertex, standard or factorial representation).

In the game Like Terms, flexibility could be evidenced by students' suggestions of more than one equivalent expression to the given expression.

Summary: The commognitive tool for characterizing players' participation in the mathematics discourse of a group while game playing

The questions discussed above are summarized in Table 4.

Characteristics of de-ritualization	Operational definitions
Agentivity	Does the group set relevant tasks and independently choose relevant procedures?
Applicability	Are the group's task and procedure based on mathematical precedents which were produced during the game (by the game design or by another player) or

	were they based on mathematical precedents which were produced before the game?
Bondedness	Does the outcome of each mathematical step in the procedure feed into the next one? Are all procedure steps relevant to achieving the desired task, or are some steps redundant?
Substantiability	Does the group justify the suggested solution?
	On their own?
	In response to others?
Flexibility	Does the group apply different procedures to solve the same mathematical task?

Table 4– operational questions to identify groups' explorative participation

4.5 Analyzing the data - second RQ

The second RQ dealt with the game design and game rules: *Which characteristics of game design promote or hinder explorative participation?*

Students' participation while playing is strongly affected by the game rules and design. Game rules determine the goal; that is, the task(s) that players need to accomplish to win the game. The rules also determine the actions that are authorized or prohibited when playing. Addressing this question was expected to clarify RQ1; that is, to determine whether the characteristics of students' participation while playing could be explained by the game design and rules.

For instance, in the game Catch the Stars students are required to use algebraic expressions to graph their answer. The task is to graph linear or parabolic lines and the procedure involves creating the graphs by typing in their algebraic expression. However, students are free to choose which algebraic expression to type, in what form (vertex, standard or factor) and to determine their limits. Therefore, I examined the game context and the potential allowed by the game design to provide and encourage explorative participation.

The analysis of the game designs in terms of providing explorative participation was based on the characteristics detailed in section 4.4.3. However, the characteristics were adjusted to the context of game design as summarized in Table 5.

Characteristics of de-ritualization	**Operational definition of game design**
Agentivity	Do the game rules and design require independent performance and decision making throughout the game?

Applicability	Do the game rules and design require the players to apply familiar routines? routines produced during the game (by the game design or by other players)?
Bondedness	Do the game rules require students to execute their mathematical move in more than one step? Do these steps have to be bonded?
Substantiability	Do the game rules and design require justifying the players' moves?
Flexibility	Do the game rules and design require applying different procedures to the same mathematical task or applying the same procedure to different tasks?

Table 5- operational questions to identify the potential of the games' promotion of explorative performance

Note although the game rules and design could hinder or promote certain characteristics of explorative participation, it was up to the players to take advantage of them. In this study, opportunities to participate exploratively were compared to the students' actual participation.

4.6 Ethics

This study was conducted during class time. It was approved by the Ethics Committee of the Ministry of Education and the Technion's Ethics Committee (approval number 2020-105). Parents of all the students who participated in this research received a written explanation about the study (Appendix 1) and signed a consent form (Appendix 2). There was no pressure to be videotaped and students who refused to participate were not penalized in any way.

Videotaping took place only during game-play time and the cameras were focused solely on the students' hands and on the game pieces (e.g., the tiles, game board and computer screen). An alternative activity was prepared in advance for students who chose not to participate in the game.

To protect the participants' privacy, no identificatory details are provided. Pseudonyms are used in this thesis.

4.7 Trustworthiness of the study

In this qualitative research, trustworthiness was ensured in two main ways according to principles set down in Lincoln and Guba (1985):

1. Prolonged engagement: The researcher was involved in every step of the study, from developing the games to using them in class. The researcher was fully acquainted with the use of all three games, and with the participants as students in the mathematics classroom. This type of participant-observer involvement contributes to a better understanding of the groups' participation in the mathematical discourse while playing. However, this level of involvement of the researcher raises questions as to her professional judgment given her high

engagement in the study (Tabach, 2011; Tabach, 2006). This is explicitly discussed in the Limitations sections (Section 6.6). However, the impact of this argument can be countered by considering the following:

a. Choice of groups to be videotaped. Cameras were placed during break time and students could sit wherever they wished and thus choose to be filmed or not. Therefore, the students rather than the teacher chose which students were filmed.

b. Choice of which data to analyze. Choosing which group data to analyze out of the full dataset was mainly based on game types, mathematical content, and the level of the class. No personal preferences were involved in choosing the data.

2. Peer scrutiny of the research project: During the analysis, discussions on the analysis process were held with at least two experts in Commognitive discourse analysis. In addition, 20% of the rounds were analyzed independently by an expert and by the researcher and the results were compared, showing that in terms of the groups' agentivity, applicability and substantiability, there was perfect agreement between expert's analysis and the researcher. However, there was 85% agreement on bondedness and flexibility. Full agreement was achieved through further discussion. In addition, this study has been presented at two conferences and a seminar in which other researchers commented and critiqued the research and helped refine the research assumptions and methods.

Another resource was my professional diary as a teacher. I am used to writing down my pedagogical thoughts about lessons and in particular, game lessons. As a researcher, I specifically kept three diaries in which dilemmas, thoughts, observations, insights, doubts and questions were noted during the entire research process. For instance, in my teacher's diary I wrote "students are very happy to see the Totem game on their tables after the break. They talked <u>a lot</u> during the game. I think it was part of the game but I can't be sure. Maybe they are excited about their Jerusalem day activity" (13.05.2018). Comments and thoughts similar to these showed me how hard it is for a teacher to implement games in class.

5. Findings

In this chapter, each characteristic is detailed separately, to address the two research questions. The answer to the first question cover possible evidence of each characteristic, as well as a demonstration of how the commognitive methodological tool was used to characterize the way students participate in mathematical discussions while game playing.

Since each characteristic emerged differently in each game, in some places examples from different games are presented.

5.1 RQ1: Agentivity

Agentivity was found in the groups' independent choices. The findings show that students' agentivity was evidenced while performing the following four tasks: (1) producing or choosing mathematical narratives and procedures by brainstorming together, (2) clarifying the mathematical task by asking questions, (3) planning the answer, and (4) executing relevant procedures independently. I consider the performance of each of these tasks as a certain *type* of agentivity. At the end of this section, an example of lack of agentivity is presented. Tasks 1, 2 and 4 were found in all games, whereas task 3 was only evidenced in Catch the Stars. Although each task manifested slightly differently in the different games due to game design and mathematical content, only one example for each task is presented.

5.1.1 Agentivity - producing and suggesting mathematical narratives and routines

This type of agentivity was found in all the games and groups. Here, it is illustrated in the Totem game, in which students produced mathematical narratives about quadrilaterals and clarified unclear vocabulary and solutions. The production of narratives included deciding which of the suggested narratives were correct and which were not.

Excerpt 6 shows a group in the Totem game that looked for quadrilaterals to match the property card of "unequal diagonals".

Excerpt 6 – producing mathematical narratives by brainstorming

#	Name	What was said [what was done] (added information)
357	Raffi	In a square they (diagonals) are equal
358	Galia	In a kite they (diagonals) are not (equal)
359	Raffi	Yes
360	Sasha	Not equal
361	Anna	Neither in a rhombus
362	Galia	In a trapezoid they are also not equal. Are they?

363	Raffi	In a square they (diagonals) divide each other and are equal
364	Galia	Right, but where? (according to the card) they are not equal
365	Sasha	But in an isosceles trapezoid they are (equal), I think so
…	…	…
399	Anna	So, in everything that is not a rectangle, square and isosceles trapezoid

The Totem game is about producing geometric narratives about the properties of quadrilaterals. The group in Excerpt 6 worked together independently to brainstorm possible narratives. Their task was to find as many quadrilaterals that matched the given property as possible. They chose to map which quadrilaterals matched the property card or not. A kite (358), rhombus (361) and trapezoid (362) were considered to correspond to the property card while the rest (399) were considered mismatches. The group brainstormed the solution together and all four members of the group discussed possible solutions. In this game the students produced mathematical narratives jointly by working together, and the students responded to each other's suggestions although one student could respond to a comment addressed by another student to a third student even if it looks a bit scattered. For instance, Sasha (365) responded to Galia (362) about the trapezoid, Anna (361) responded to Galia (358) and so on. While brainstorming, the students in the group chose independently to listen to each other and gradually found the solution.

5.1.2 Agentivity - clarifying the mathematical task by asking questions

Sometimes while brainstorming or solving the problem, students raised specific questions which were addressed by the rest of the group members. This was typical of all the games and groups.

In the example below, the group was discussing which quadrilaterals match the property card of "diagonals are not perpendicular". In this excerpt a rectangle is discussed.

Excerpt 7 – group's agentivity of setting mathematical questions and answers

#	Name	What was said [what was done] (added information)
55	Galia	But, how is it that the diagonals are not perpendicular in a rectangle?
56	Sasha	In a rectangle they (diagonals) are not perpendicular
…	…	
61	Anna	Why? (are diagonals not perpendicular in a rectangle)
62	Sasha	If they it were perpendicular it (the rectangle) would have been a square

In Excerpt 7, Galia and Anna ask “how” (55) and “why” (61) a quadrilateral such as a rectangle does not match the property card. Sasha, another member of the group, provides an answer by applying the routine of inclusion. He claims that a square is a type of rectangle with perpendicular diagonals (62).

This type of agentivity emerged as students raised relevant questions when they were not sure about the brainstormed narratives and other group members independently provided them with the answers. As can be seen here (56), others asked Sasha to clarify her response (61). Students took responsibility for their answers (56) and shared their mathematical narratives (62). All these suggest that students exercised agentivity.

5.1.3 Agentivity - planning the answer

In Catch the Stars, it was most evidenced that brainstorming that was part of students' planning stage. Brainstorming was part of planning a solution before performing a procedure.

In excerpt 8, Shalev and Ohad have just reached their fourth screen and are brainstorming two options for solving the problem (a parabola graph and a linear graph). However, they also identify important mathematical features that should be considered as part of the planning strategy. This planning is relevant to each of the suggested solutions in the brainstorming stage.

Excerpt 8 – agentivity of planning in the Catch the Stars game

#	Name	What was said [what was done] (added information)
148	Shalev	A **parabola** is needed to be added here [pointing towards (0,0) point and slides his finger in a parabolic line from stars at point (-2,0.8) to (2,0.8). See figure 1]
149		It's needed to be added **here** [pointing towards (0,0) point and slides his finger in a parabolic line from stars at point (-2,0.8) to (2,0.8). See figure 4]

150	Ohad	No, no, it’s fine (means there is no need to create a parabola)
151	Shalev	It needs to be added **here** [slides his finger at an imaginary parabolic line that connects the three stars from, (-2,0.8), (0,0) to (2,0.8)] something that will hold (the ball)
152	Ohad	Even a **straight line** [**points** at the line y=0 with his finger], **straight line**, **Y zero** (would hold the ball)
153	Shalev	But then it (the future linear graph) wouldn’t **grab this point** [pointing towards the star which on point (-2,0.8)]

The group’s task is to find a graph that will be part of the trajectory of the balls on their way to catching all the stars (see figure 4). Students independently suggested different procedures as part of their brainstorming stage. Shalev suggested creating a quadratic function (148,149,151) while Ohad suggested creating a linear function (152).

However, Shalev stressed the fact that they should plan their answer according to important points (stars) on the screen. The choice to look for anchors (points in this example) to guide them to the chosen solution shows their independence while solving the mathematical problem.

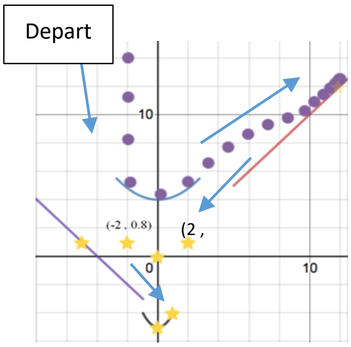

Figure 4- unsolved screen

5.1.4 Agentivity - executing relevant procedures

This type of agentivity was present in the continuation of the same episode of the Catch the Stars game shown above. Shalev and Ohad chose one solution and wrote its quadratic function (Excerpt 9) to create the planned graph. Students in this game set the task of planning the relevant graph and then used the procedure of typing the relevant algebraic expression to achieve their task. They also independently decided to modify the algebraic expression if needed (Figure 5).

Excerpt 9- agentivity of choosing the relevant procedure

#	Name	What was said [what was done] (added information)
159	Shalev	(erases y=0) Y equals ahhhh (thinking) 0.5 [types $y = 0.5x^2$]
160	Ohad	Yes
161	Shalev	Ahhh 0.4 [changes 0.5 to 0.4: $y = 0.4x^2$]
162	Shalev	0.2 [mistakenly erases the entire expression when intending to change 0.4 to 0.2]

In Excerpt 9 the group applies the procedure of creating an algebraic expression that suits their plans (159). They move on to modify their expression on the fourth screen according to their desired outcome (figure 5 and 6). The group goes through the procedure independently by trial and error. In Figures 5 and 6 modifications of parameter A and the function's limits can be seen.

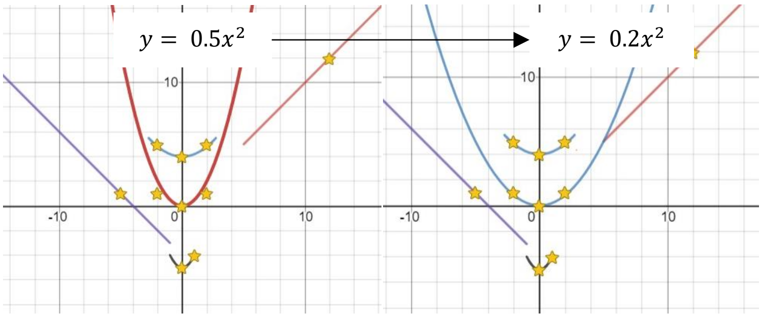

Figure 5- modification of the first parabola (in red, on the left) to the final parabola (in blue, on the right)

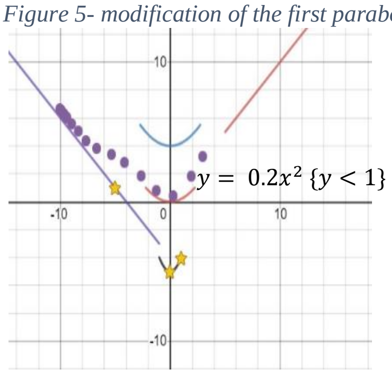

Figure 6- last modification of the parabola in Figure 5 by adding limits

This example demonstrates agentivity by choosing a procedure. Students execute their brainstormed mathematical narratives while keeping an eye on their planned anchors. The whole procedure described here (in Excerpt 9, Figures 5-6 and Table 6) was chosen independently by the group itself and executed by them. The tasks and procedures enacted by the students in Excerpts 8 and 9 are summarized in Table 6.

Task	Procedure
Plan the trajectory of the ball and plan a general graph	Watch the ball's movement on the screen. Consider where to place the graph (148-152). Take notes of the places the ball should go through (148,149,151). Consider where to place the graph to hold the ball (151, 153).
Find the algebraic expression that matches the target graph	Shalev suggests graphing a parabola (148) from point (-2,0.8) to (2,0.8) with a minimum vertex at (0,0). Ohad suggests using a linear graph (152) y=0. Ohad suggests y=0 and Shalev writes it. Assessing whether it would solve the problem by imagining the ball's trajectory (158). Shalev suggests a parabola instead of y=0 and writes $y = 0.5x^2$.
Adjust the written expression to get to the solution.	Trial and error to find the right aperture of the branches: Shalev writes $y = 0.4x^2$ Shalev writes $y = 0.2x^2$ Looking at the Y axis and assessing the boundary of the parabola: Shalev writes {y<1}

Table 6- tasks and procedures enacted by Shalev and Ohad while solving the fourth screen

5.1.5 Lack of agentivity

There were some rare episodes in which groups disengaged from the mathematical task. In these cases, they either called on the teacher for guidance (which was less common) or skipped to the next round (Excerpt 10). Giving up on trying to solve the problem indicates lack of agentivity.

Excerpt 10- Shalev and Ohad in their sixth round

#	Name	What was said [what was done] (relevant information/clarifications)
189	Teacher	(the teacher is heard in the background) If a slide [screen] is too difficult you can skip it and return to it later.

Excerpt 10- Shalev and Ohad in their sixth round

190	Shalev	[after launching the ball for the third time] Wait, maybe we should skip it. I'm not up to it
		…
196	Shalev	We did. Well, let's skip it (slide 6). Halas (enough)
197	Ohad	What is this? (on the screen) No, what is it?
198	Shalev	It's, what is it? this shit [presses an arrow to skip to slide 7]

In Excerpt 10, the students do not brainstorm possible solutions at all. They consider the screen to be weird (198) and do not even start looking for a procedure to apply. The only question they ask "what is this ?" (197, 198) is not math-oriented and therefore no answer or intent to look for a mathematical answer is produced.

Thus overall, agentivity was found in different types of situations. All types shared groups' responsibility for the mathematical solution. In all cases, the groups' actions were directed at obtaining the mathematical goal of the round.

5.2 RQ2: Characteristics of the game design that promote or hinder agentivity

In terms of game design, I examined whether the game rules and design require independent performance and decision making throughout the game.

The players mostly followed the game rules and responded to its design while playing. In terms of agentivity, some game rules and design can, however, promote or hinder different types of agentivity. In general, students' independence is enhanced by games rules, which provides them with turns; in other words, time for independent performance. In Totem and Like Terms, each student must perform separately while in Catch the Stars performance is done in pairs. This may suggest that the students who agreed to play would execute a move on every turn. That is, students who agreed to play in fact agreed to take responsibility for their turns.

In the Totem game, brainstorming in the group could be enhanced by the rule that one property card is valid for all players in the round. Thus, executing moves related to the property card is based on group consensus that takes shape in the brainstorming stage conducted by all the students.

In Catch the Stars, brainstorming and planning could be enhanced by the requirement to find the optimal trajectory for the ball. Thus, students needed to pay attention and discuss the critical points the ball must pass through.

In Catch the Stars, the ability to skip a screen hindered the groups' agentivity. In Excerpt 10 both students quickly skipped without being encouraged by the game to cope with the problem. The teacher is heard in the background saying "If a slide [screen] is too difficult, you can skip it and return to it later" (189) right before the group skipped the screen. This is a result of the game's objective of solving as many problems (each screen is a problem) as possible and the technical option of skipping without solving the problem on the screen. In Like Terms, by contrast, there was no special rule or design encouraging agentivity except the turn element discussed above.

Thus overall, the basic characteristic of turns may promote students to take responsibility for their game-playing. Certain common components such as a shared card in Totem or shared screen in Catch the Stars might enhance agentivity in games. On the other hand, the ability to skip a turn can hinder agentivity as players relinquish their turn without trying to solve the mathematical problem.

5.3 RQ1: Applicability

In this study, three types of applicability were defined: (1) applying former narratives and routines, (2) applying narratives and routines learned from another student while playing, and (3) applying narratives and routines learned from the game design while playing. The first and second types of applicability were common to all three games, but the third type of applicability was exclusive to the Catch the Stars game because of its design.

5.3.1 Applicability - applying former narratives and routines

Applicability has to do with students' application of relevant procedures and tasks while solving the mathematical challenge. Applicability engages students' former experiences of doing mathematics that the students see as "the same" or similar enough to be applied in a somewhat distanced context (sometimes even a new context), for example, typing equations to generate a certain graph in game context.

For instance, Odel and Offir created the parabola $y = x^2 - 17$ that they wanted to move horizontally (see Figure 7). As can be seen in Figure 7, they needed to move the blue parabola's graph to the left (transforming it into the red parabola). To do so they applied a former procedure of creating a parabolic graph by writing the vertex form (Figure 7). They reminded each other of the form of the algebraic expression $y = (x - p)^2 + k$ and the sign they should choose when moving the vertex horizontally by saying "…the X ahhm do it here (pointing to where to write the brackets) the other brackets…" or "If we want to move it here (to the left) it's plus (the sign before parameter p)".

In this example, students' precedents about the vertex form were successfully applied (as shown in Figure 7 in the blue rectangles bolded on the left, marked by the arrow) to achieve the mathematical outcome of creating the target parabola.

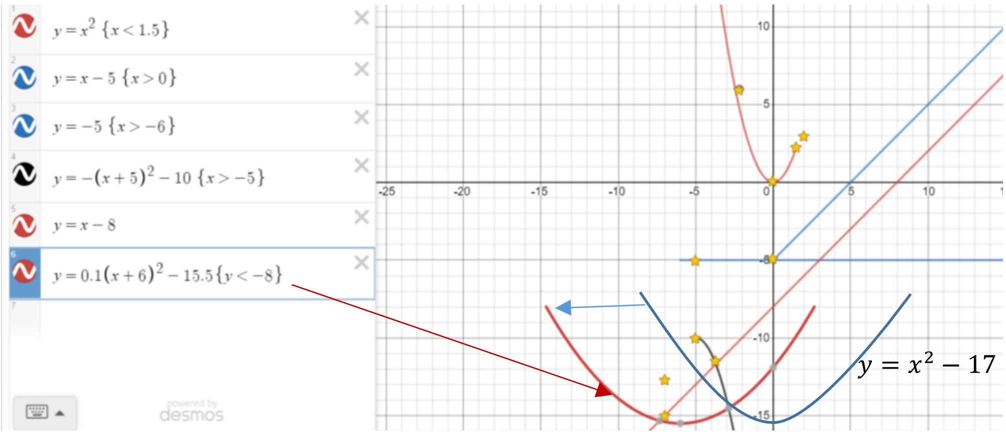

Figure 7- Vertex representation in the fourth screen

5.3.2 Applicability - applying narratives and routines learned from another student while playing

This type of applicability is demonstrated by the game Like Terms, as seen in Excerpt 11. In this game students were asked to apply a procedure of adding terms until an equivalent expression was created.

In the following excerpt (which is partially shown in section 4.4.3 Excerpt 4) Ron applied Dan's procedure of adding like terms to create an equivalent expression.

Excerpt 11- when one student applies another student's mathematical procedure

#	Name	What was said (what was done)
29	Ron	I don't have I don't have (nervously)
30	Dan	Do (pick) eight ($8x^2$) turn it [takes a tile with -3x, and flips it and separates it] this three (3x), plus nine (sounds like he is thinking out loud) and here [turns to Ron] do six ($6x^2$)[picks $6x^2$ and separates it] find me another 2 ($2x^2$) like this (i.e.,x^2). Here two [picks a $2x^2$ tile and lays it near the $6x^2$] you already have one (of the terms in the target expression) you see you already have one. That's how it has to be done
31	Ron	What? (surprised) I understand. Here, let's take two, we'll take two (funny unclear voice) [picking a card of x squared and putting it back. Later on he picks 3x and 5x]

Ron does not apply any procedure for combining like terms. We may assume from his reaction in Excerpt 4 (23) "I don't have I don't have anything that relates to eight ($8x^2$) or nine ($9x$) and not to seven (7)" that combining like terms according to Ron involves looking for the exact missing term written on the board in his tiles; for instance, by matching $8x^2$ on the board with $8x^2$ on the tile. His interpretation of the task is not how the game is played since all the tiles in the game had coefficients between -6 to 6. However, after Dan shows Ron how to create the term $8x^2$ by using tiles $6x^2$ and $2x^2$ in Excerpt 11 (30), Ron starts to apply the same procedure. Notably, Dan did not explicitly explain the procedure of combining like terms to Ron. Rather, he demonstrated the game "move" (picking the tiles and adding the numbers). Ron then looked for tiles to create 9x. He started by picking 3x+5x. Later on, he finished by adding x. Thus, in this excerpt Ron applied the combining like terms procedure that he learned from Dan during the game.

5.3.3 Applicability - applying narratives and routines learned from the game design while playing

In Catch the Stars, all the graphs on the right side of the screen had matching algebraic expressions on the left side (as shown in Figure 8). Students could easily identify which algebraic expression matched a graph since they were in the same color. In addition, when clicking on a graph, the matching algebraic expression was bolded (and vice-versa). In the following example, one group of students applied the creation of a horizontal parabola typing an adequate equation that was based on an existing horizontal graph and equation in the game setting. Excerpt 13 demonstrates how students interacted with game design and were encouraged by it to explore new expressions.

Excerpt 12– Shalev and Ohad discover the horizontal parabolic graph

#	name	What was said (what was done)
321	Ohad	….and why did he do that here (points to the algebraic expression of the horizontal parabola)
322	Shalev	Ahh in order to draw this one [parabola] (moving his finger along the existing purple parabolic graph] why it seems to be me? [the creator of this challenge] looks very familiar [clarifying- Shalev did not create this challenge]
324	Shalev	Look! (turning to Ohad and pointing to the algebraic expression of $x = 0.02(y+150)^2 + 250$) X equals Y squared
…	…	
333	Shalev	Ahhh I don't remember how it works (how to create a horizontal parabolic graph) I don't remember it (after typing $x = 0.02y^2$)
…	…	
336	Ohad	No, move it like a normal parabola
337	Shalev	Ahhh (agreeing with Ohad) right, right, but I don't know if it works. I've brought it (the graph) up right? (typing $x = 0.02y^2+$))
…	…	
343	Shalev	No (referring to the + he typed) minus minus minus (points at the X axis on the -150 and type $x = 0.02y^2 - 150$)
…	…	
347	Shalev	Now it has to be moved up in ahhh (typing brackets and the value of $x = 0.02(y-150)^2 - 250$)

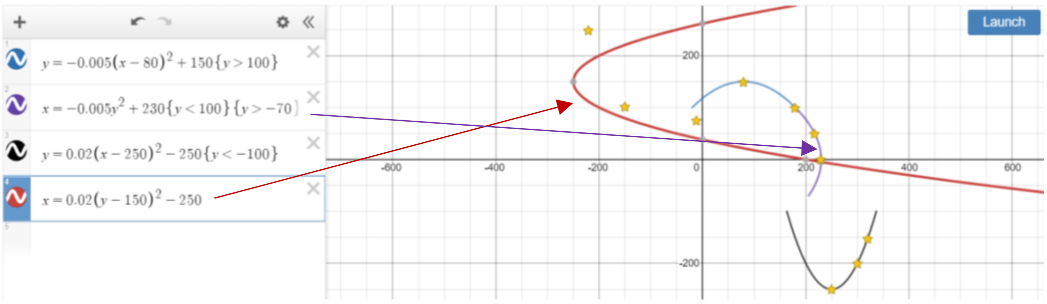

Figure 8- Ohad and Shalev's tenth screen

Shalev and Ohad are examining the properties of the horizontal parabola (purple parabola marked by the purple arrow in Figure 8) (321, 322). Shalev notices its algebraic expression "look! (at the given data) X equals Y squared" (324). Both Shalev and Ohad decide to apply this new expression in their solution. Shalev declares that he is familiar with this type of graph but cannot remember how to apply it "don't remember how it works" (333). As a result, they mimic the existing algebraic expression on the screen and type it into the left-hand table. They decide to apply familiar procedure of a standard representation "move it like a normal (standard) parabola" (336).

Shalev transforms precedents of vertical movement into horizontal movement by changing parameter C (337). He reflects on the sign and decides to use the value -150 (343). Then, to move the graph up, he changes the standard representation into a vertex form and adds what is known to him as parameter P to create a vertical movement of the graph (347). Shalev combines relevant familiar procedures with the given data on the screen so he can produce a new mathematical narrative. In this case, the game design encouraged students to explore new algebraic expressions.

Thus overall, the mathematical games in this study derived strongly from mathematical content. Mathematical content was integrated into the game design and rules such that every move to be executed involved mathematical applicability. Some design features such as the background screen in Catch the Stars were part of the students' procedure in their solution. This is discussed in detail in the following section.

5.4 RQ2: Characteristics of game design that promote or hinder applicability

In terms of game design I inquired whether the game rules and design require the player to be familiar with previously learned procedures to play successfully. All three games required a certain familiarity with mathematical routines and narratives. However, not all games required a specific procedure to solve the mathematical problems presented in the game. In Totem, no particular procedure other than matching the property card to the quadrilaterals was required. As a result, all the players suggested mathematical answers in the form of a final mathematical narrative such as "in a square, all sides are equal". Game design did not encourage students to perform a mathematical procedure to reach the solution. The search for procedures was encouraged by the groups' mathematical discussions.

In both Catch the Stars and Like Terms games, the game design encouraged students to apply specific procedures. In Like Terms the rule was to physically collect tiles that are added up (mathematically) to form a new expression. In Catch the Stars students were asked to type algebraic expressions and set their limits to gradually create the target graph on the screen.

Catch the Stars is the only game that helps students with the procedure through its special design as explained in the third example in the previous section.

Overall, the mathematical games in this study derive from mathematical familiar narratives and routines. Previously learned mathematical narratives and routines are integrated by the game design and rules such that every move to be executed mathematical applicability in the game context. In addition, the analysis of RQ1 in section 5.3.3 suggests that applicability was also evidenced by the way the Catch the Stars screen was designed, which made it visible for students to visually track the way existing graphs (the background) are formed by algebraic equations. Students explicitly demonstrated their applied procedures in two out of three games (Catch the Stars and Like Terms) through rules and the design that required students to do so on every move and every turn.

5.5 RQ1: Bondedness

The analysis suggested that bonded routines were present in rounds of all three games. These bonded routines differed in mathematical content from one game to another. The findings also suggest another type of bondedness: bondedness of global routines. I use the term *global routines* to define a sequence of smaller routines performed by the players. Different groups playing the same type of game (Totem and Catch the Stars) performed a pattern of routines with a structure that was similar and was repeated almost every round. I call this pattern a global routine. Within each global routine, students performed bonded sub-routines. In the following example bondedness in both global routine and regular sub routines are demonstrated.

5.5.1 Bondedness- Global and sub- routines

In this game, I found macro-bondedness, the bondedness of a “global routine”, and micro bondedness which defines the tasks and procedures in the sub-routines comprising the global routine. A global routine in both groups of students who played Catch the Stars consisted of three steps: planning, writing an algebraic expression and modifying it. The sub-routines constitute the steps to carry out the global routine (Figures 9 and 11).

All three steps in the global routine were relevant to the solution and the outcome of one step fed into the next one. The end of the planning step determined which algebraic expression the students wrote down. The written expression and its type (for example vertex or standard representation) determined which type of modifications would be made (for instance modifying parameter a,b,c or a,p,k). The last modification, if all steps were correct, led to the solution.

Every macro-step consisted of bonded subroutines (with new tasks and procedures). The planning routine usually included procedures such as launching the ball and watching its trajectory, finding the star points on the coordinate plane, and creating an imaginary path for the ball to Catch the Stars. The writing procedure involved choosing which algebraic expression to write (quadratic or linear, standard, factored or linear) according to the data on the screen. The modification routine included changing parameters and launching the ball for the last time. The global routine and its sub-routines were found in both groups and in every round (figure 9).

In this section Excerpts 13-15 focus on Odel and Offir's global and micro (sub) routines as presented in Figure 9.

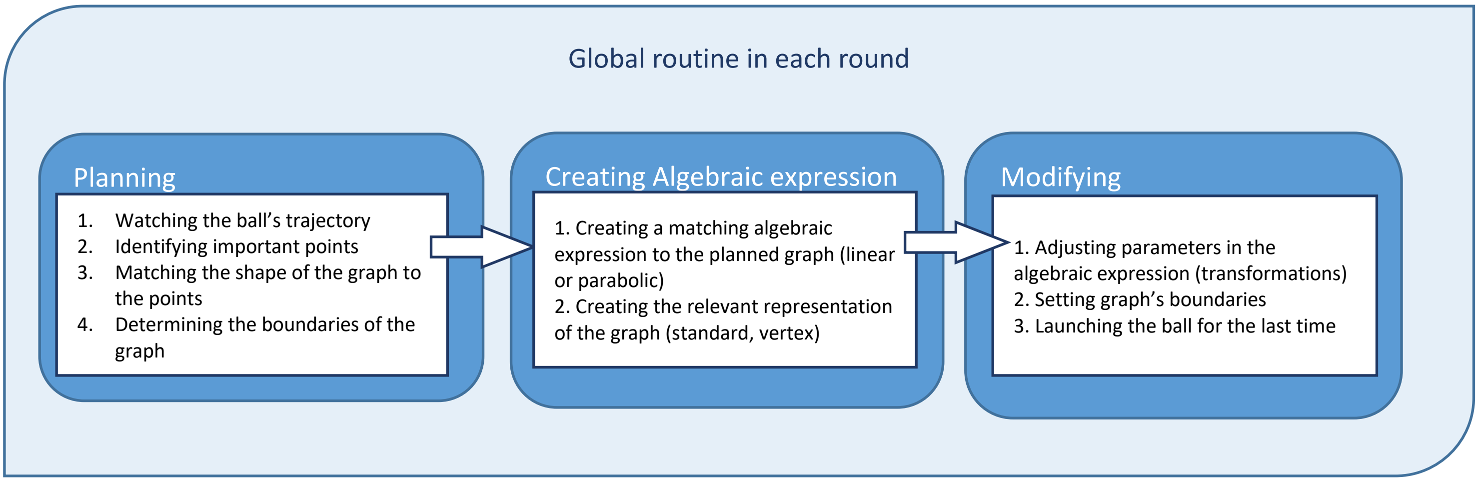

Figure 9 - Bondedness of global routines and sub-routines

Each of the following excerpts (13-15) exemplifies a single step in the global routine. All three excerpts refer to the same group and the same mathematical problem in the game Catch the Stars (Figure 10).

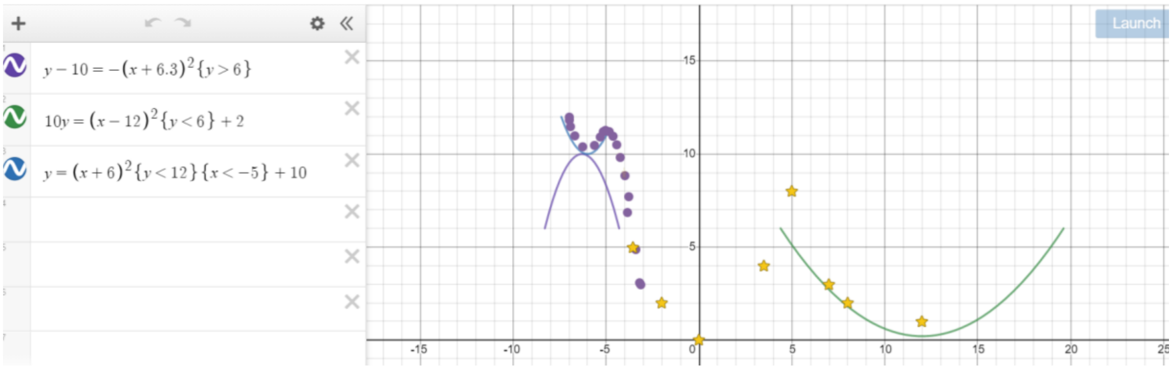

Figure 10- Odel's and Offir's launched ball in the second round

First step in the global routine and its subroutines

In Excerpt 13, a group of two students (Odel and Offir) are solving the first screen in the game Catch the Stars.

Excerpt 13 – Bondedness in Odel and Offir's planning for the second round.

#	Name	What was said [what was done] (added information)
1	Odel	[the ball was launched] ok so this (the right branch of the purple parabola in figure 12) sh- should be continued
2	Offir	Why should it be continued and not [unclear turns 1-2 are spoken at the same time]
3	Odel	Just continuing it [silence for 10 seconds]
4	Odel	Here [pointing to the limits of the existing algebraic expression of the purple parabola {y>6}]. So I say let's not cut it at six let's cut it let's say here [pointing to (0,0) point] and then we'll add to it a line (linear graph). Do you get what I'm saying?
	…	(After launching the ball again)
8	Odel	Yes, actually something like that [sliding her finger from the right end of the purple parabola in a parabolic way to the left edge of the green parabola] and then it's like bouncing them (the balls) here (pointing to the stars on the green parabola). So, we need something that will pass through the coordinates of the origin.
9	Offir	A regular parabola (meaning $y = x^2$) and we just open it (expanding the distance between its branches), it's not… (complicated)

In Excerpt 13, the students are trying to decide which graph will solve the screen. This is the first step in the global routine (planning). Two suggestions are bonded: Odel's first suggestion leads to Offir's second suggestion (this is part of their brainstorming). The steps of the sub-routines according to Excerpt 13 are presented in Table 8:

#	Steps in sub-routines	bondedness
1	The ball is launched (1)	Since the ball falls along the imaginary x=-3, the ball's trajectory should lead it towards the right part of the screen to catch the rest of the stars.
2	Odel suggests continuing the purple parabola up to (0,0), after checking its current boundaries (2-3)	Since the new boundaries leads the ball no farther than (0,0) point, another graph should be added
3	Adding a linear graph from (0,0) to the left edge of the green parabola (4)	After the initial plan is created, it should be carefully checked
4	Launching the ball again (7)	Since the critical change of direction occurs at the (0,0) point (from moving down to moving up)
5	Focusing on the (0,0) point (8)	Since this is a critical point, it should be the turning point of the planned graph
6	Offir suggests a parabola $y = x^2$ with a greater distance between the branches (9)	Since the vertex of $y = x^2$ is (0,0)

Table 7- bonded steps of planning according to Excerpt 14

Table 7 shows that each step leads to the next one. By launching the ball students can visually follow its trajectory on the Cartesian system and students then consider a possible path. After observing the data on the screen (the launched ball, stars (points), graphs and their algebraic expressions), Odel makes her first suggestion. Her suggestion has to do with the (0,0) point, the existing boundaries of the purple parabola and a linear graph that should pass through point (0,0). Once they have a plan, they launch the ball again. The purpose is to focus on the (0,0) point since it is a critical point that is expected to lead the ball (if it gets there) to the green parabola. After watching the second launch, Offir suggests a parabola ($y = x^2$) as a possible solution. The parabola's minimum vertex is at (0,0) so the ball's trajectory on this parabola can lead the ball to the green parabola after catching the stars on the left. They now go on to the next step where they write the algebraic expression (Excerpt 14).

Second step of the global routines and its subroutines

In this step, the students type the algebraic expression that best suits the last output of the planning stage for the first time.

Excerpt 14 – writing the algebraic expression of Odel's and Offir's planned graph.

#	Name	What was said [what was done] (added information)
9	Odel	… do [means type] ahhh Y equals zero point zero two X squared [asking Offir to type]

The step of planning, which led to the use of a basic parabola $= x^2$, is considered a sub-routine of the macro-routine's second step. Odel guides Offir on which expression to type, suggesting the function $y = 0.02x^2$ (9). This function is bonded to two elements from the planning steps: the graph should pass through point (0,0), and the distance between its branches should be wider than the original parabola of $y = x^2$.

Once the expression is written, its graph automatically appears on the screen. This was where students went on to the last step of modifications (excerpt 15).

Third step of the global routines and its subroutines

The graphed parabola of $y = x^2$ is much wider than the group expected. Therefore, mathematical transformations need to be done. The transformations are shown in Excerpt 15.

Excerpt 15 – Odel’s and Offir’s modifications for the second round.

#	Name	What was said [what was done] (added information)
11	Offir	Isn’t it more open (the parabola’s branches) than it (should be)
12	Odel	Do three (type 0.3 instead of 0.02), the previous three (0.3) was good [referring to their solution in the previous round].
13	Odel	No, maybe it’s one and a half type 0.5?
14	Offir	0.5 (Offir types). More open (a wider distance is needed)
	…	
18	Odel	0.4
19	Offir	Let’s launch [they launch the ball and the ball gets stuck in the intersection of the new and existing parabolas]
20	Odel	Ohh, we need to cut it (new parabola’s branches)
21	Offir	Here ahh just a moment
…	…	(in these turns of speech students are following the existing algebraic expression of boundaries which are written on the left side of the screen. They decide to apply it)

28	Odel	Yes, how can we shorten (the graph) [silence for a few seconds] that's how we're supposed to shorten (the graph)[pointing at the written boundaries] maybe you'll do Y smaller than… ahh five [Offir types]…
29	Offir	Let's do launch [Offir launches the ball and it catches all the stars]

The expression $y = x^2$ from the second step went through the following modifications according to Excerpt 15:

1. $y = 0.02x^2$ is too wide so the students transform it into $y = 0.3x^2$ (12).
2. $y = 0.3x^2$ is still too wide so they transform it into $y = 0.5x^2$ (13-14).
3. $y = 0.5x^2$ is too narrow so it is transformed into $y = 0.4x^2$ (15,18).
4. The students launch the ball and it gets stuck between the graph of $y = 0.5x^2$ and the given purple parabola (19).
5. Setting boundaries (by applying the form of {y $</>value$} from the given boundaries in other expressions) {y>5} in turn 25. The students see that the vertex is not graphed (21-28).
6. They modify boundaries to {y<5} (28).
7. They launch the ball and win the round (29).

Although there is some redundancy (in Excerpts 14 to 16), we can see how one step is tightly bonded to the other. Redundancy in this case was not considered low applicability since the students are building the procedure as they play. Hence, a smooth sequence of steps is not expected. However, students are expected bond one step to the other.

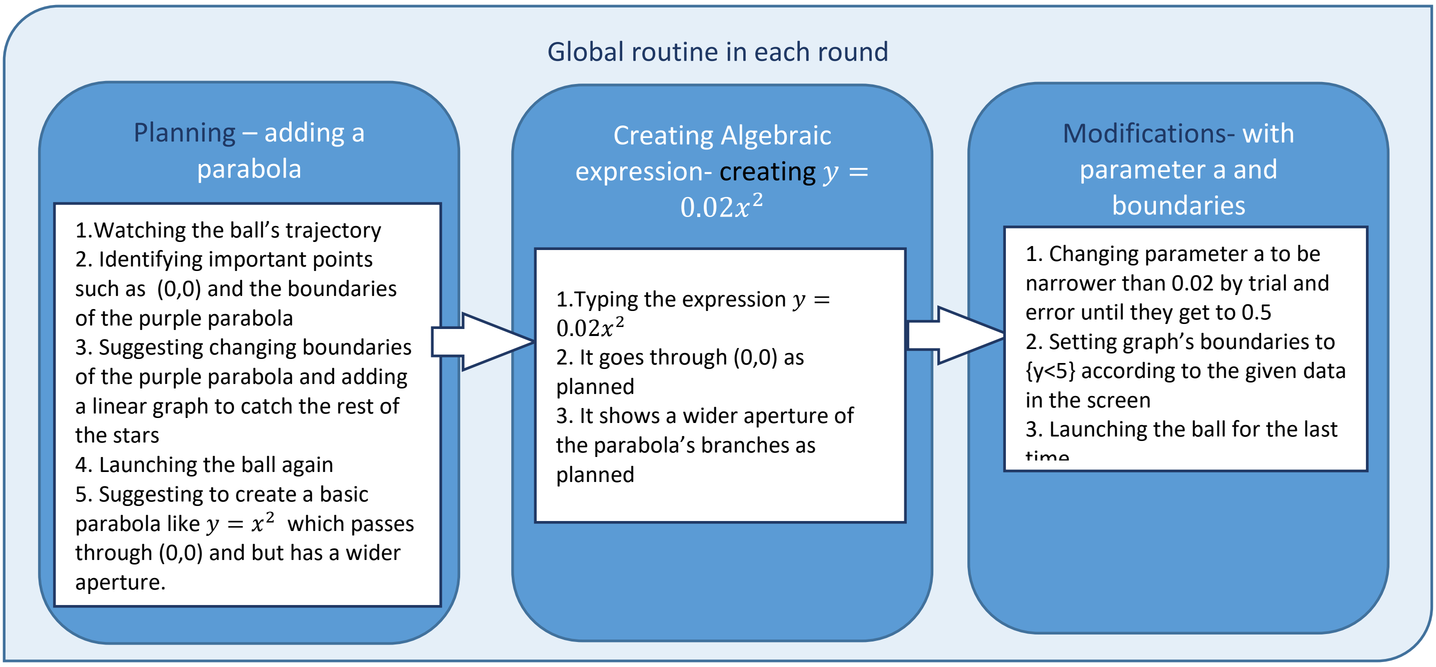

Figure11 - global and sub routines in Odel's and Offir's second round

All the bonded steps in this round (examples 13-15) are summarized in Figure 11.
Global routines were also evidenced in the Totem game. Naturally its steps were different since the game and content are different.
In general, the analysis showed that each of the Totem groups repeated the following global routine of producing a solution in the same manner. Here are the global steps of Totem:

1. Reading a property card. The output of this step is that a certain property is explicit to all group members.

2. Suggesting solutions. The input of a known property resulted in students suggesting quadrilaterals that fulfill this property. These suggestions could be considered as narratives of the form "in *quadrilateral x, property y*" (e.g., in a square, all sides are equal). The output of this step is a list of specific quadrilaterals that fulfill the given property.

3. Agreeing on a mathematical solution. The list of quadrilaterals performed in the former section is now open to negotiation. Students discuss and justify the produced narratives to reach an agreement.

4. Moving the plastic chip over an appropriate quadrilateral to make progress in the game. Narratives endorsed by the group enable students to agree which ellipse they can move their plastic chip to during their turn.

In both games (Catch the Stars and Totem) all the sub-routines were part of the global routine. In Like Terms, on the other hand, routines that were used to solve the mathematical problem were less complex, as can be seen in the following section.

5.5.2 Bondedness- routines that are not part of a global routine

In this example, Ron's performance in the game Like Terms is reported (Ron did not say anything but rather simply performed by moving the tiles). Ron executed routines to create an equivalent expression to the given expression: $-9x^2 + 10x - 7$ in the round. In this example, only Ron's explicit actions were analyzed since he did not talk. This short episode lasted 5 minutes and 17 seconds. The episode starts at 6:49 and finished at 11:23..
Ron repetitively looked to combine each term: x squared, x and numbers separately, in every round. Then he created each part in the target expression individually (there is no fixed order). When finished, Ron assembled the all parts to display his solution. Every time Ron picked a tile, he created a “domino effect”. Each tile he picked (apart from the first one)

depends on the previous tile and influences the choice of the next tile. This is the way to combine expressions into a final equivalent expression.

To do so, in this example Ron disassembled $-9x^2 + 10x - 7$ into three parts $-9x^2, 10x\ and - 7$ and dealt with each part separately. His first attempt was to solve for 10x but he left it unsolved (00:09:31-00:09:57). He moves on to -7.

When Ron picks a tile (with a relevant term), it affects the following tiles he picks . When he starts to create $-9x^2$ he picks $-3x^2$ (00:10:11) which means that he needs to add $-6x^2$. He picks another tile of $-3x^2$ (00:10:17) which leads him to look for the last term of $-3x^2$ (00:10:18). In this example, the last tile must be equal to or greater than $-3x^2$.

Every step in Ron's sub- tasks of creating the three terms of $-9x^2, 10x\ and - 7$ separately are bonded. Every time he picks a tile, his future possibilities are narrowed.

Overall, in this section, examples of global routines and their sub-routines were presented as well as routines that were not part of a global routine. Students' math-making was enacted to move closer to finding a solution. That is, students' procedures and steps were chosen to get to the goal which was always immersed in mathematical meaning. Since there was usually group work involved, the steps were bonded but some were redundant. In the next section, aspects of game design will be discussed in relation to bondedness.

5.6 RQ2: Characteristics of game design that promote or hinder bondedness

To answer the 2nd RQ in relation to bondedness, I followed the guiding question of *do the game rules require students to execute their mathematical moves in more than one step? Do those steps have to be bonded?*

All three games had a clear goal of winning. Players are expected to calculate every move carefully. Every move should bring them closer to the goal. Since it is impossible to win by executing one move, each move needs to serve the next move until a win is accomplished. Thus the game-playing strategy must be bonded by default, but what about the mathematics in the game?

The game design of Catch the Stars provided students with three steps they needed to go through to win. The first is to launch the ball and identify the path (graph) that has to be created. The second step is to key in the algebraic expression and function boundaries. The third step is to launch the ball again to determine whether any corrections need to be made. If no corrections are needed players go on to the next screen (mathematical challenge). As shown in Section 5.5.1 students usually went through those steps at the global level.

At the micro level, the game design provided two types of feedback. Every time a student changed a parameter in the algebraic expression or boundary the computer provided immediate changes in the graphs. These changes served as output the students relied to win. In addition, students were allowed to launch the ball as many times as needed. This was another form of immediate feedback provided by the computer.

The game design of Like Terms provided students with the opportunity to solve the mathematical problem in steps. Although the numbers and parameters on the tiles ranged from -6 to 6, the terms in the expression on the board were either higher than 6 or lower than -6. In other words, the players had to combine at least two tiles to solve the mathematical problem. In addition, all the expressions included three types of terms: x squared, x and numbers. Therefore, every player needed to go through at least six steps to solve the mathematical problem in every round.

In the Totem game the rules required the players to match the property to the quadrilateral. Every step on each quadrilateral had to be aligned with the property card of at the same round. Though students created a routine of playing in the round it was not a direct consequence of game rules or design.

Note that game design can only provide students with the *opportunity* to create the equivalent expression in a few steps but students need to perform a bonded routine to participate in an exploratory manner.

Overall, the games in this study, in particular Catch the Stars and Like Terms required more than one step (from three steps to at least six) in order to complete a turn. This may prompt students to bond procedure steps, and therefore enhance bondedness.

5.7 RQ1: Substantiability

In general, the findings indicated two types of substantiability: (1) convincing others, and (2) explaining to others. This section illustrates each type of justification.

5.7.1 Substantiability- convincing others

In the following excerpt, three out of four students in the Totem game agree that a rhombus matches the property of "two pairs of opposite equal sides". However, one student (Sasha) disagrees. The other students in the group try to convince him. They keep justifying their narrative until they all reach an agreement.

Excerpt 16- substantiability to convince a group member.

#	Name	What was said [what was done] (added information)
184	Raffi	Does a square count (as having two pairs of opposite equal sides)?
185	Galia	Yes, yes, yes
186	Raffi	Cause everything is equal (sides in a square) and parallelogram.
187	Sasha	Also a rhom(bus) no, yes… (hesitation)
…		
196	Sasha	In a rhombus the opposite sides a(re), are not equal
197	Galia	In a rhombus all sides are equal
198	Sasha	But that doesn't mean that they (sides), like necessarily the opposite ones are equal, (but) the adjacent (sides) are (equal).
199	Anna	but all the sides are equal
200	Galia	But all the sides are equal in rhombus it is like the square.
201	Raffi	Listen if these (sides) were four, four, four, four then four equals four, four equals four.
202	Sasha	Well. So. O.k.

Substantiation is evidenced here when players try to justify that a rhombus is a possible solution to the property card, in order to convince Sasha. They do so in three different ways:

The first justification by Galia and Anna is based on mathematical logic of the form 'if… then…' (197, 199), if all sides are equal then the opposite will be equal as well. Sasha is not convinced and by saying "necessarily" (198) he appears to suggest there might be a case where all sides are equal, but the opposite sides are not. It is also possible that Sasha's interpretation of the card focused solely on opposite sides that are equal and they cannot all be equal.

The second justification is an analogy (200) to a square. Galia asks Sasha to think of a rhombus in the same way he thinks of a square (two quadrilaterals with four equal sides). She relies on group's earlier agreement in turns 184-186 where she and Raffi briefly discussed and agreed that a square matches the property and Sasha did not oppose that mathematical conclusion.

The final justification is a specific example by Raffi (201) who gives the value *four* to each side of the rhombus. Raffi's justification may relate to a specific case implicitly mentioned by Sasha earlier by saying "necessarily" (198). Raffi compares the values of the opposite sides "…then four equals four, four equals four "(201). Since the values are equal, the sides should be equal as well.

5.7.2 Substantiability- explaining to other group members in Catch the Stars

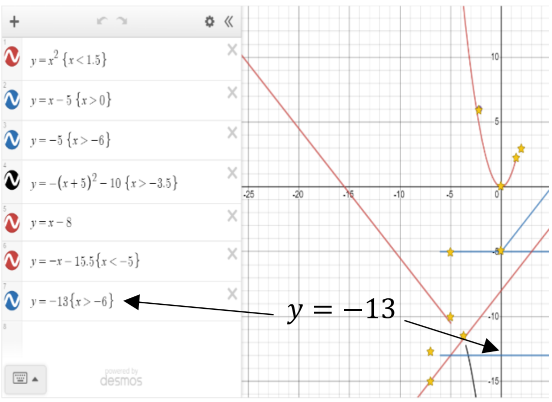

Figure 13- solution of screen 3 by Ohad and Shalev

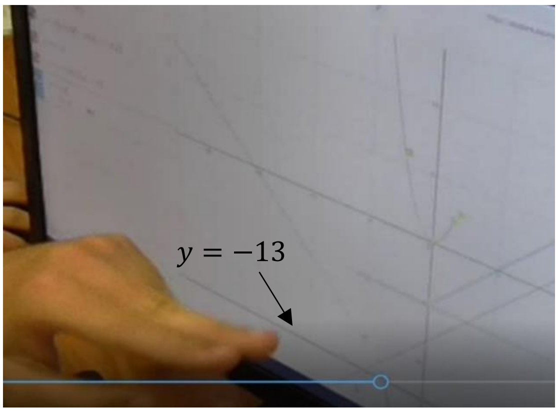

Figure 12- Shalev's hand gesture while justifying

Excerpt 17 shows the way in which justification emerges as an authentic need to obtain an explanation about a mathematical narrative in the Catch the Stars game. The goal of the justification is to help a friend (as opposed to answering a teacher's direct question or convincing a student). In Excerpt 18, Ohad asked Shalev to explain the procedure of how to decide with which value (x values or y values) limits should be determined.

Excerpt 17- Shalev's and Ohad's substantiability in the third round

#	name	What was said (what was done)
129	Ohad	It's Y or X now?
130	Shalev	X X
131	Ohad	How do you know?

Excerpt 17- Shalev's and Ohad's substantiability in the third round

132	Shalev	Cause you need it to be Y (y=13), you can't shorten (vertically) the Y. Cause the Y is, you shorten [illustrate vertical movement- see figure 12] everything (all values) beneath (a certain y value) to [moving his hand vertically to show what is beneath the value point of the boundary] or above (a certain y value) [moving his hand vertically above the point], right
133	Ohad	So (thinking)
134	Shalev	And this X is to the sides [demonstrating a horizontal movement with his hand]
135	Ohad	Ahhh (insight intonation) great
136	Shalev	You can't shorten the above or below when it is about one linear line (constant line where the slope equals 0). Aahhm [start typing the boundaries {x>-6}] so X is greater than…

Shalev's justifications were related to the specific mathematical situations but also referred to a more general view. For instance, he not only regarded the constant function but also the general idea of directions (horizontal and vertical) when deciding to set the boundaries by the X or Y values.

Ohad clearly formulates a request for an explanation of the boundary procedure (129, 131). Ohad asks Shalev to explain how to decide which procedure is appropriate to set the boundaries (setting by the values of X or Y). This is a continuation of the original question Ohad asked before this excerpt "Is it Y or X how do you distinguish between them?" (112). Ohad's request assumes that Shalev knows (112) the answer.

Their task is to set boundaries to a constant linear function y=-13 (see Figure 13). Shalev first refers to set of boundaries as a procedure that "shortens" (132) the graph. He means that only some of the values (range) need to be graphed. Shalev then refers to directions: (1) vertical direction – "beneath… to… or above" (132) shortening as opposed to (2) horizontal direction– "to the sides" (134) shortening (figure 13). Then he justifies his procedure (setting X values) by relating to the constant function of y = -13 and explaining that the vertical direction is not relevant "You can't shorten above or below when it is about one linear line (constant) … so X is greater than." (136). Shalev hints without explicitly saying that there is only one y value (13); therefore, it is impossible to define a range by Y values "you can't shorten (vertically) the Y" (132). As a result, the use of the X values is required – "so X is greater than" (136). Shalev's justification takes advantage of the existing mathematical situation of a constant function on the screen. Boundaries of a constant function (a linear graph that is parallel to the X axis or unified with it) which are defined by Y values might conceal the graph but will not graph a segment of it.

Shalev’s justification is accompanied by hand gestures on the screen (Figure 12). When talking about directions he simultaneously moves his hand according to the directions he mentions and the location of the $y = -13$ function (as shown in Figure 12).

In this section, justification was triggered by questions from a group member. The two types of justification were found in all three games and in all groups.

5.8 RQ2: Characteristics of game design that promote or hinder substantiation

The games in this study have no rules that explicitly require players to substantiate their moves, mathematical narratives or routines. However, this surprising fact did not hinder students from seeking justifications. As can be seen in the previous section, students indeed justified their mathematical narratives and routines. This may have to do with the fact that games were played in groups where differences of opinion were likely to take place. Next, I describe some implicit game design elements that may provide students with the opportunity of justifying their routine.

Substantiation in the Totem game

The Totem game rules state that the same property card is relevant to all players until the round ends by changing the property card. As a result, everyone shares the same property card in a round. Hypothetically, there are at least two ways of playing. One is to supervise every player’s move, by checking the quadrilaterals chosen by the players in every turn. Another way is by reaching an agreement among the group members about quadrilaterals right after the property card is read. Though the students were familiar with the rule that if a player executed a wrong move (mathematically chooses the wrong quadrilateral) the rest of the players are allowed to cancel the player’s move and move her backwards. Most still chose to play by the second option. This decision increased the likelihood they would try to convince each other.

In both groups, a group member suggested to play by the first option (and take advantage of the rule mentioned above) in an implicit way and met harsh social refusal. An example was presented in Excerpt 3 (Section 4.4.1). In Excerpt 3, Raffi suggests that the players should stop being nice to each other saying “From now on, let's be less nice “ (143) and refuses to share mathematical narratives (165, 169). Anna responds harshly by calling him a "shit" (172). Raffi apologetically says that he was "just kidding" (173) and continues to play by sharing the mathematical narratives and routines.

Substantiation in Catch the Stars

This game is designed to give instant feedback in both graphing the expressions and launching the ball unlimited times. There were instances in which students did not have to convince each other but rather tried out the solution and watched the computer's feedback. Therefore, immediate feedback from the computer through students' trial and error process (while the writing and modification task) seemed to lessen the number of students' justifications as they could literally see if their procedure led to a solution as planned.Overall , the game designs and rules did not require substantiability (except as an implicit aspect of the Totem game). By contrast, the findings in Section 5.2 show that group dynamics (which is beyond the scope of this study) may enhance explorative participation of substantiability even when the game rules do not require it. In addition, as found in Catch the Stars, immediate feedback can hinder substantiability. The feedback shows whether a suggested solution works. If it does, students tend to skip the mathematical justification.

5.8 RQ1: Flexibility

Flexible implementation of routines was less common in all three games. In the rare instances when it did take place, it was manifested differently across games.

5.8.1 Lack of flexibility

Flexibility in Totem game

Students' routines were rarely flexible. In cases in which flexibility was identified, it was part of the students' justifications. An example was shown in Section 5.7.1 Excerpt 16. In this excerpt, students suggested three different procedures (logic deduction (196, 199), analogy (200) and a specific numerical example (201)) to convince Sasha that a rhombus has two pairs of opposite equal sides. Flexibility was a result of the students' need to convince another group member that their solution was right.

Flexibility in Catch the Stars

Flexibility was less common in both groups and was mainly displayed in the planning routine where students tended to suggest different graphs for the ball's future trajectory . Once they agreed on a plan, they stuck to one procedure.

An example can be seen in Section 5.5.1 Excerpt 13 where Odel and Offir suggest two different ways to solve the problem presented in the second round. Odel suggests a combination of an adjustment of the purple parabola and an additional linear graph (4). Offir suggests adding only one parabola (9). They agree on Offir's suggestion and continue with the mathematical procedure which corresponds to their plan.

Flexibility in Like Terms

In this game the procedure is always the same (adding like terms together). However, there are different terms that can be combined to create the required expression. This can be seen in two ways, as different correct solutions among students, or different solutions by the same student. Since I focused more on Ron's game, I can only describe the flexibility of two solutions to the same expression.

When students lay down 36 tiles (6x6) they create random possibilities for future solutions in the round. For example, when Ron tried to create 10x out of 5x,4x, x but did not find a tile with x (00:10:32-00:10:37) he changed his terms according to another possibility that he found – 5x and 5x (00:11:17-00:11:31).

Every time a student picks a tile it affects the rest of the tiles needed to combine the remainder of the expression. Thus, students were constantly changing the solution according to the tiles they picked. Sometimes, different equivalent expressions were created by the students. This could be considered flexibility to some extent since this game was played in the Telem class where students are struggling and underachievers. Nevertheless, flexibility was rare.

Overall, since flexibility was rare and most of the time a procedure was lacking, no types of flexibility were identified.

5.8.2 RQ2: Characteristics of game design that promote or hinder flexibility

The potential opportunities for flexible participation in the games were examined by asking whether the game rules and design require applying different procedures for the same mathematical task or whether they require applying the same procedure to different tasks.

In Totem, the best strategy to win the game is to find as many matching quadrilaterals as possible. However, there was no rule about how to do so and students were not required to apply different procedures to do so.

In Catch the Stars, the rules did not require solving the mathematical problem in more than one way. However, the game design provides students with the opportunity to explore more than one solution by enabling them to create many graphs. In addition, students were told that there are many solutions to the same problem.

In Like Terms the procedure was always the same as well as the task; namely, adding like terms to create the equivalent required expression. Thus, the game design may hinder students' flexibility since it encourages them to stick to the same procedure.

Overall, in all three games, the procedures were more or less fixed by the game rules and design.

6. Discussion

This study set out to examine two research questions: (1) *what are the characteristics of students' explorative participation while playing games in middle school mathematics classrooms?* (2) *which characteristics of game design promote or hinder explorative participation?*

The findings corroborate previous studies on digital and mathematical games showing that while playing, students are highly engaged with mathematics and are encouraged to stay on task and be more involved in problem solving during the game (Byrne, 2017; Deater-Deckard, Chang & Evans, 2013). Section 6.1 provides a closer look at the ways in which students talk about mathematics and to characterize their participation while playing. (as will be elaborated in section 6.1).

In addition, examining pedagogical aspects of game design as conceptualized through the theoretical Commognitive lens in Section 6.2, made it possible to analyze how game rules and design promote and hinder opportunities for explorative participation (as elaborated in section 6.2).

The findings to both research questions lead to new recommendations on the teacher's role in classroom game play, as detailed in 6.3.1. Suggestions for a better design of mathematical games are presented below. The methodological tools based on the commognitive framework that can be added or combined with existing tools appear in 6.4. The whole research process was a meaningful journey for me as the teacher, developer of games and researcher. This process is summarized in the reflective section 6.5. Finally, limitations are detailed in section 6.6.

6.1 Addressing RQ1 – Students' participation in game playing

The findings show that overall, students' participation was characterized mainly by agentivity (Section 5.1.1), applicability (Section 5.3.1) and bondedness (5.5.1). Each characteristic manifested in different ways. This implies that the games provided the students with opportunities to participate exploratively in different ways. Further research should investigate the extent to which explorative participation varies from one game to another. No clear instances of flexibility were found in this study (Section 5.4.2), and substantiability was rare (5.7.1). Further research is required to better understand the reasons for these findings beyond explanations rooted in game design (6.2). What can we learn about students' explorative participation in games according to these findings?

Agentivity

The most prominent characteristic of students' explorative participation found in this study was agentivity. The findings suggest that playing mathematical games promotes group independence by providing them with the opportunity to choose which procedures and tasks to follow or discuss. Active and independent learners are the core of Constructivist theory (Noemí, & Máximo, 2014; Gee, 2005). The findings confirm that participants in each group decided independently on which mathematical procedure to focus and when to further discuss a mathematical narrative.

Students exercised agentivity in almost every round of every game examined in this study. Group participation included a shared mathematical discussion where combining each student's participation and response to another group member created a whole group identity of agentivity. The results indicate that students decided what to do and how to do so (Section 5.1). In the Totem game, students produced mathematical narratives together, as a group, before moving on to actual play (5.1.1). In Catch the Stars, mathematical solutions were planned during group discussions (5.3.1). In Like Terms Dan's agentivity gave Ron the opportunity to participate in the game and start combining like terms. Interestingly, all groups in all games took responsibility for the mathematical problems that were presented in all three games. This may be another form of engagement which is described in so many studies as simply staying on task or a state that is related to students' affective, cognitive and behavioral participation in games (Deater-Deckard, Chang, & Evans, 2013). Agentivity may also be explained by Vygotsky's (1976) Zone of Proximal Development, since these three games were tailored for students' mathematical level in this study (Hamari, Shernoff, Rowe, Coller, Asbell-Clarke, & Edwards, 2016).

The different types of agentivity in game situations described in this study (5.1) show that there are many opportunities during a game (from planning the solution to executing the mathematical procedure) where students can execute agency.

Applicability

Games provide students with the opportunity to explore familiar concepts. Moreover, group discussions provide students opportunities to share solutions and produce solutions (mathematical narratives) jointly. The findings showed that students not only applied formerly routines, but also applied new mathematical procedures that were presented to them during the game for the first time. The findings suggest that students applied routines during the game by watching the game design unfold (Catch the Stars) (5.3.3) or by asking other group members to explain mathematical narratives and routines (in all three games) (section 5.3.2).

Applicability based on game design elicits students' involvement and are aligned with the Constructivist paradigm which states that learners should be involved and participate actively in their own learning process (Noemí, & Máximo, 2014). Applicability based on asking questions may best be accounted for by the socio-cultural paradigm where communication (as evidenced here in the games) is a crucial part of participating in the learning process (Vygotsky, 1976; Gee, 2005). Further research is needed to investigate how students learn new mathematical subject by playing in class.

Future research should also explore how teachers can encourage students to apply new mathematical routines for the first time while playing And, the teacher's role in exposing students to new narratives through games.

Bondedness

Bondedness was found not only in the students' discourse (sub- routines) but also in the broader perspective of doing mathematics (global routines). Students' routines performance were bonded in the global procedures (in Totem and Catch the Stars) and most sub-procedures (in all three games). For instance, in Catch the Stars, students performed three macro-procedures in each round (planning, creating expressions, and modifying them) to solve their mathematical task (section 5.5.1). Every macro-procedure was executed by many sub-bonded routines. The repeated global routine seemed to help the students organize their steps in the solution procedure. By doing so they broke down mathematical problem solving into smaller tasks and then again into sub-routines. This is aligned with evidence from other studies showing that students improve their logical thinking by creating steps to solve the mathematical problem and develop skills needed in problem solving (Henry, 1973; Siew, Geofrey, & Lee, 2016; Orim, Ekonesi & Ekwueme, 2011; Way, 2011; Wiersum, 2012).

Substantiability

Substantiation was found in all three games (Section 5.7) in one of two cases. Either students substantiated their narratives when trying to convince the group, or a group member asked for an explanation of the mathematical solution. Interestingly, during substantiation, the word 'mistake' (or cognates) was never uttered. This finding suggests that students' authentic need was to discuss mathematical gaps and differences in narratives and routines rather than emphasize mistakes. Students did not label any mathematical solution as a "mistake".

Thus, in an educational system that is still mostly based on error evaluation (like tests), games may provide students with the opportunity to participate in a different way when using mathematical discourse (Way, 2011; Wiersum, 2012). In addition, students' types of substantiation as were found in this study, encouraged students to responsively ask about and clarify mathematical issues that need to be fine-tuned.

One explanation for the presence of students' substantiation is rooted in Flow Theory (Csíkszentmihályi, 1990, Csíkszentmihályi, 1996) which refers to the way a person is strongly engaged with an activity to the point that time goes by without noticing. The person is immersed into the activity and is focused on the process. Although the game does not require it, students who immerse themselves in the game expressed the need to justify, explain or ask about mathematical tasks and procedures. Further study could look for the links between the Commognitive theory and Flow theory in games (ibid). It is also possible to analyze the way group dynamics influence substantiation and flow (Camerer, 2003). In addition, it would be interesting to analyze how students substantiate when it is required by the game, as opposed to social requirements.

Flexibility

Lack of flexibility should be further investigated. The findings indicated that students tended to perform the same procedures without trying to look for new ones.

6.2 Which characteristics of game design promote or hinder explorative participation?

Overall, considering both research questions, applicability, agentivity and bondedness characterized students' explorative participation in all three games. These characteristics were also embedded in the games' design and rules. Flexibility was not encouraged by the game design and was rarely found in the groups' mathematical discourse. However, substantiability, which was not required by the game rules or design,was more commonly found in the groups' participation. This may imply that there was an interplay between the design of a game and its rules, and the characteristics of groups' explorative participation. The students followed the game design and rule requirements. Interestingly, students carried out some of the missing requirements, as described below.

Agentivity

Some basic game rules may promote students' agentivity. Games in this study (and in general) ask students to take an active role by playing in the form of a turn. A turn is a time and space allocated by the game rules where the player is required to take action. The game cannot be continued until a player ends her turn. In the three games, taking an action in a turn always included a mathematical decision as well as a game decision. When playing in pairs each student had a technical role (one was typing and the other was in charge of the mouse pointer) which may have promoted their joint involvement in the game and mathematical decisions. In other words, the turns may have provided the students with an explorative starting point. Afterwards it was up to them to decide how active and involved they wanted to be.

The findings showed that in certain game conditions, the goal of winning may have interfered with the students' agentivity. In the game Catch the Stars, the game design that enables students to skip a screen enabled students to avoid coping with mathematical challenges. In this case, that students relinquished their mathematical agentivity in a round to better pursue their goal of winning the game. In contrast, students in Totem pursued mathematical solutions (matching only the correct quadrilaterals) even if it limited their game progress. In Totem, accepting all quadrilaterals as a solution (just to win the game) takes the motivation out of winning. If the rules offer an opportunity to skip over a tough issue, students may skip to win. However, students in this study respected the rules and although it might have brought them closer to winning the game, they stuck to the rules.

The basic design step of allocating a specific time in which players should act seems to enhance all types of agentivity.

Applicability

All the games were based on the students' mathematical curriculum. Specific mathematical topics could be seen in the design, such as tiles with terms on them, screens full of graphs and algebraic expressions or property cards. This design may have helped the students to relate to specific topics and performing certain mathematical procedures.

In Catch the Stars, transparency in the screen set (background) design exposed students to mathematical content that consisted of the algebraic expressions that were graphed on the screen as its background. Such visible content may encourage students to produce mathematical narratives and procedures and explore.

The game design in this study was tailored to the students' previous precedents at least partially. However, the context of the mathematical content was slightly different as compared to textbooks and worksheets. The students were encouraged to apply former routines appropriately during the game. Though the mathematical problems were open ended and could be solved in more than one way, they were based on the same familiar mathematical routines. Therefore, the players were prompted to apply the same procedures without reaching the same solutions.

Bondedness

Bondedness was promoted by the game rules and design. From the game perspective, the students need to move towards the goal from their first game position at the beginning of the game. Hence, they dive into more and more specific steps that eventually have to be bonded to achieve triumph in the game. Every game's ultimate optimal strategy is to win with no redundancy at all. Games sequences of actions such as launching the ball, typing and launching again (in Catch the Stars) may have provided a global game routine that could then be broken down into sub-routines. These game rules and design encouraged the students to solve the mathematical problems in steps. Further research should investigate if there is any connection between game routines and mathematical routines during mathematical games.

In some cases (Catch the Stars and in Like Terms), bondedness could be monitored by students' actual steps of typing the algebraic expression of the function or picking certain tiles in a particular order. These steps were available to the researcher through the game design. Thus, those games in which the required mathematical procedure was translated into game elements such as tiles and typing functions could enhance the students explicit math-making . When designing a game, creating such elements may encourage bondedness in students' participation.

Substantiation

Substantiation was not required by the game rules. There was no explicit rule that encouraged students to substantiate their mathematical solutions in any of the games. Hypothetically, students could have executed their move, based (or not) on their mathematical decision without justifying it throughout the game. In addition, computer feedback hindered the possibility of substantiation since students could simply see for themselves if their solution was right (section 5.8). The findings with respect to the different types of substantiation suggest that it occurred as a result of social interaction among group members. Further research should investigate the differences between substantiation triggered by game design and triggered by social interaction alone. Requiring substantiation as part of game design can easily be achieved by adding a rule that encourages students to substantiate, such as "if a player can justify her move, she may take two more steps".

The findings show that substantiation was part of social interaction, as reported elsewhere in the literature (Vankúš, 2005; Gough, 1999; Gee, 2008, Gee, 2011). Hence, perhaps implicit rules (such as a shared card in a round) may enhance social interaction and as a result encourage students to justify their mathematical procedures.

Flexibility

Flexibility was not required by any of the three game rules or design. In Totem, students were expected to find more than one solution to each problem posed by the property cards, but they were not encouraged to do so via different procedures. Catch the Stars had more than one possible answer but to solve the screen the procedure was constantly repeated; namely, typing algebraic expressions. While multiple game strategies can offer an efficient way of playing, the students need only one way to solve the mathematical problem in the game to proceed according to the game strategy. In other words, there was no strategical advantage for players who found more than one way to solve the problem.

The l ack of a flexibility requirement in the game rules may hinder students' flexibility. There are explicit ways to add rules that can require flexibility such as "you may combine like terms by using any of the four operations".

6.3 Insights from the findings when combining both research questions

Several insights can be derived from the findings when considering both research questions together. These relate mainly to the teacher's role while students participate in game-play (6.3.1) and how to design mathematical games for students (6.3.2).

6.3.1 What can a teacher do to promote explorative participation through mathematical game playing?

This study showed that students are engaged with the mathematics while playing. However, groups of players may address different mathematical issues (within or outside the mathematical content of the game) according to the group's dynamics and level of familiarity with the mathematics embedded in the games. Thus, teachers should monitor students' comments while playing in class and keep track of their mathematical discussions. This is a familiar didactic technique, which is often called "monitoring" (Stein, Engle, Smith, & Hughes, 2008). Monitoring helps the teacher assess students' participation in mathematical discourse and to prepare upcoming lessons or discussions.

Thus, teachers should hold a class discussion at the end of game play lessons. It provides students with the opportunity to share and discuss mathematical solutions and narratives that were not necessarily part of their group game. In addition, while discussing mathematical solutions and narratives with the whole class, the teacher can encourage flexible participation and justifications in the class discussion after the game. It is important to do so at the end of game session and not in the middle since students are engaged in the game and stopping them every now and then is likely to disengage them.

6.3.2 Designing the game

Using mathematical games as defined in this study provided students with explorative opportunities to participate. In particular, agentivity and bondedness derived from the rules of the games themselves. High applicability in students' participation was the result of the students' familiarity with procedures before the game but at other times procedures that were learned during the game. This shows that explorative participation in game playing may provide students with new narratives or procedures not only for practice (Friedlander, Markovits, & Bruckheimer,1988). Further research should be conducted to identify the limits of how and what new mathematical content can be learned for the first time through games.

The findings showed that when players are exposed to relevant mathematical narratives and procedures via the game design (such as in Catch the Stars) they may be encouraged to participate exploratively and try to apply them in the game. Thus, game designers should try to incorporate samples of solved problems in the game design (even on the game box covers).

The most basic component that emerged from this study was players' interactions. This requires at least two players in a game. Having two or more players promotes interaction among players (students). This type of interaction is the essential element for social learning (Vygotsky, 1976; Vygotsky; 1978) which may in certain situations encourage explorative participation.

When designing a game, teachers should consider ways to actively incorporate flexibility and substantiation in game rules to enrich students' opportunities to participate. This could be done by adding a rule for example that players can ask their opponent to justify their mathematical decision at any moment of the game. If the answer is wrong, the opponent loses his or her last move but if it is correct, the opponent is entitled to play an extra turn.

6.4 Affordances of the Commognitive tool presented in this study

Some studies have noted that there is no model (Gee, 2011) or methodology to analyze students' learning while playing. To date, researchers have mostly used Flow theory (Kiili, Lainema, De Freitas, & Arnab, 2014; Kiili, K, De Freitas, Arnab, & Lainema, 2012) with operational pre/post interviews and questionnaires (Kiili, 2005). There are some frameworks such as Constructive Alignment (Aleven, et al., 2010) along with designing and analyzing framework of integration of the three components (Kalmpourtzis & Romero, 2020) introduced in Section 3.4 to primarily guarantee that students' participation in the game will adhere to the learning objectives.

The findings here suggest that providing students with opportunities for explorative participation can be one of the goals offered by games in mathematics classrooms. Adopting the Commognitive conceptualization of de-ritualization, which characterizes the various characteristics of students' participation on the continuum from more ritualistic to more explorative participation, made it possible here to study various aspects of students' participation while playing games. The Commognitive framework provides researchers with operational tools that can be easily adjusted to different games (digital and non-digital) as long as they involve students' discussions while playing games with a game definition as suggested in this study.

This study s used Lavie, Steiner and Sfard's (2019) definitions of the characteristics of de-ritualization and applied them to the world of educational games in the field of mathematics. In so doing I found it necessary to construct a special methodological tool for examining students' participation in game playing. Hence, this study contributes not just to the literature on face-to-face games but also provides a methodological contribution, since the tool provides ways to examine students' modes of participation during the game.

The Commognitive tool also related to game design, which was based on operational questions about the characteristics of de-ritualization. However, this conceptual tool shifted the focus from players' performance (or students' participation) to the design itself and to the game rules. This tool may help teachers anticipate what type of participation is required by a game. This is a different aspect of game evaluation since it is strongly related to specific pedagogical concerns, and in particular, to the pedagogical goals of students' doing of mathematics while playing. I believe this tool can be combined with other evaluations of game design.

6.5 Limitations

This study took place during school days in actual daily mathematics classes. Therefore, the mathematical content was determined by the school curriculum and could not be changed. A major drawback was the time limit. Each class is 45 minutes long and could not be changed. Therefore, when students did not finish the game, I could not ask them to keep playing. In addition, the students had been in my classes for the previous two years. Therefore, they were exposed to mathematical games long before this study took place, and are used to play in mathematics lessons. It is possible that students who have never played mathematical games would react differently.

Being the researcher, teacher and game developer required constant awareness about the way I interpret situations and students' discourse. For example, it took me a while to understand that students' flexibility emerges from students' mathematical needs in the game but is not required by the game. As a developer of the games, I assumed that since there are many ways to solve the mathematical challenges in the game, students would actually apply them. Lack of flexibility surprised me and changed the way I have invented games since then. To overcome this limitation, I wrote down my thoughts as a teacher and researcher in a journal and constantly shared my thoughts with other researchers (see section 4.7).

Since the lessons were videotaped before the study took place, specific lessons could only be chosen from a corpus of existing videotapes. In other words, the lessons were not designed for this research and the database was limited.

6.6 Researcher's perspective

Adopting a researcher's point of view in addition to that of a teacher took time. Going over students' excerpts and videos gradually revealed the way students participated in mathematical discourse while playing. As a teacher, I believe that mathematical games should be used in class because they can be tailored to students' mathematical needs in that they enable social learning and help students master mathematical narratives and procedures. However, every time a new game was played by the students, I had some concerns. Most of them had to do with the opportunities for explorative participation that students are provided with when playing. Many questions were raised. Does the competitive aspect of the game interfere with my students' mathematical discourse? How much time is spent on mathematics during the game and in what ways? Do students ask (initiate) mathematical questions when they do not know an answer or do they simply give up? What do they ask and how do they assist and explain to each other? What is my role as a teacher while game lessons take place? Should I join them or just observe? How can games be improved to promote students' independent participation in mathematics lessons? And above all, how do students actually participate in mathematics discourse when playing? Although this question was answered in detail in this research (Section 5.3) some of the other questions need to be addressed in further studies.

As a teacher, some of the data captured my attention though they were not the focus of this study. For instance, students did not label any mathematical solution as a “mistake”. They patiently explained their mathematical opinion until they reached an agreement. When they could not reach an agreement, they called on the teacher for assistance. The way students grasp failure during game is termed being ‘free to fail’ in the literature. Students are not threatened by the possibility of being wrong (Stott & Neustaedter, 2013). Mistakes are referred to as “graceful failure” (Plass, Homer, & Kinzer, 2015). In this research the contrast between students’ tolerance in their mathematical discourse and their critical and blunt comments when discussing game rules or strategies was surprising to me. One student even labelled the aggressive reaction of another student “game mode”. This was not the focus of this study, and further research on subjectifying could be conducted (Nachlieli, Levy & Heyd-Metzuyanim, 2021). As a teacher, I was concerned by the way students at times treated each other while playing and how they expressed their opinion about other students’ solutions.

Another key finding was students’ high engagement in the games. Specifically, they seemed calm and at ease while talking about mathematics. As a teacher, I feel that the hardest part of my role is to engage students in mathematical doing. I saw that students were laughing, joking and having a good time. Some of them even commented on this explicitly by saying “I’m having fun”. This could be part of a future study on playing mathematical games and the impact of mathematics anxiety. Studies show that students' engagement with mathematics changes their attitude in a positive manner towards the subject (Bragg, 2006; Alanazi, 2020; Lee, 2014).

As a developer of educational games, all three games were upgraded according to the findings. The modifications included rules that encourage more flexible participation and substantiation. Such changes should be studied as well to better assess their promotion of students’ opportunities to learn.

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8. Appendix

8.1 A latter of explanation about the research to the parents and students

חתימה דיגיטלית

תאריך: ______________

שנת הלימודים תשע"ט/תש"פ

הטכניון

מכון טכנולוגי לישראל

להורים שלום רב,

הנדון: <u>השתתפות בנכם/בתכם במחקר בנושא 'שימוש במשחקים מתמטיים ללמידת מתמטיקה'</u>

במסגרת לימודי המגיסטר בטכניון בפקולטה להוראת המדעים והטכנולוגיה, אני חוקרת את מאפייניה של הוראה המבוססת על משחקים מתמטיים בחטיבת הביניים. המחקר מתבצע בהנחייתה של ד"ר עינת הד מצויינים.

כחלק מהפיתוח המקצועי שלי כמורה למתמטיקה אני מצלמת לעיתים שיעורי מתמטיקה באופן מלא או חלקי על מנת שאוכל ללמוד ולשפר את יכולות ההוראה שלי. הצילום נעשה בכפוף לאישור התלמיד ואישור הצילום שמסרו ההורים לבית הספר.

לצורך מחקר זה אבקש לבצע פעולות אלה:

1. ליצור עותק מצילומי הווידאו בהם נטלו חלק התלמידים במסגרת שיעורי המתמטיקה בשנת הלימודים הקודמת. הצילומים נערכו כחלק מהפיתוח המקצועי שלי כמורה למתמטיקה (בהסכמת ההורים הנוגעים בדבר) וכעת, במהלך השבתון (איני מועסקת בשנה זו בבית הספר) אני מבקשת לבחון מחקרית בעזרת צילומים אלה באיזה אופן מומשו מטרות ההוראה בשיעורים בהם נעשה שימוש במשחקים מתמטיים.

2. להעביר לתלמידים שאלון עמדות בנוגע להוראה משחקית במתמטיקה. השאלון יועבר בתום יום הלימודים בתיאום מראש ומילויו מתוכנן לארוך עד לרבע שעה.

3. לקיים ראיון של התלמידים, תוך תיעודו באמצעות הקלטה קולית, כדי להבין לעומק את עמדותיהם כלפי השימוש במשחקים מתמטיים. הריאיון ייערך במסגרת פרטנית על ידי בלבד בספריית בית הספר בתום יום הלימודים. במהלך הריאיון תישמר זכותו של המרואיין לפרטיות מבלי להפריע לצוות בית הספר להשגיח עליו כנדרש. הריאיון מתוכנן לארוך עד לרבע שעה.

הנתונים ייאספו באופן המזהה את הנבדקים בפני צוות המחקר לצורך הצלבת המידע שייאסף מאותו תלמיד ועל אודותיו באמצעים השונים. ברצוני לציין מספר נקודות חשובות נוספות:

(א) המחקר הותר לביצוע על ידי לשכת המדען הראשי במשרד החינוך, בכפוף לתנאים המוצגים במסמך ההיתר מטעמה (העתק של ההיתר נמסר להנהלת בית הספר, ואפשר לעיין בו לפי דרישה).

(ב) המנחה של עבודתי ואנוכי התחייבנו על כל אלה:

- צילומי הווידיאו, ההקלטות הקוליות וכל יתר הנתונים המזהים שייאספו במסגרת המחקר ישמשו למחקר זה בלבד.

- מלבד צוות המחקר, לא תותר לאף גורם גישה לנתונים במתכונתם הגולמית (אף לא לצוות בית הספר או להורי התלמידים הנבדקים).

נספח 1

- צילומי הווידיאו, ההקלטות הקוליות וכל יתר הנתונים המזוהים שייאספו לצורכי המחקר יישמרו בקבצי מחשב המוגנים בסיסמה הידועה רק לי ולמנחה של עבודתי,

- אשמיד את עותקי צילומי הווידיאו שאצור לצורך המחקר ואשמיט לצמיתות את אפשרות זיהוים של הנבדקים מיתר הנתונים שייאספו מיד עם תום העיבודים הנדרשים לצורכי המחקר, ובכל מקרה לא יאוחר מהתאריך 31.12.2025, במועד המוקדם מבין השניים.

- פרסום ממצאי המחקר יבוצע באופן שלא יאפשר לזהות את הנבדקים או את מוסד החינוך שבמסגרתו נאספו הנתונים.

(ג) זכותו של כל הורה וילדו להחליט שהתלמיד לא ישתתף/ייכלל בפעולות שלעיל, מבלי שהתלמיד ייפגע בכל דרך בעקבות החלטתו או החלטת הוריו. כמו כן זכותם של התלמידים להפסיק את השתתפותם במילוי השאלון ובריאיון באמצע מבלי שייפגעו בדרך כלשהי בעקבות החלטותיהם. זכויות אלה יובהרו לתלמידים עצמם לפני תחילת הפעולות המחקריות.

אם אתם מעוניינים לקבל מידע נוסף על תכנית המחקר, ניתן ליצור עימי קשר במייל: paulatelem@gmail.com או בטלפון שמספרו 0544236458.

<u>אודה לכם מאוד אם תסכימו להכללת בנכם/בתכם בפעולות המבוקשות שלעיל, תחתמו כאות לכך</u> על כתב ההסכמה המצורף למכתב זה ותחזירו אותו למזכירות בית הספר, בהקדם האפשרי.

בברכה,

פאולה לוי

נספח 1

8.2 Approval of the chief scientist in the Ministry of Education

חתימה דיגיטלית

מדינת ישראל
משרד החינוך
לשכת המדען הראשי

עמוד 1 מתוך 3

ירושלים, 11 יולי, 2019
11 יולי, 2019
תיק **10537 רו**

היתר להפעלת התערבות ולאיסוף מידע על אודותיה לצורך המחקר בנושא

"שימוש במשחקים מתמטיים ללמידת מתמטיקה"

עורכת המחקר: גב' פאולה לוי

ההיתר בתוקף החל מהתאריך הרשום לעיל ועד לסיום שנת הלימודים תשפ"א בלבד

במסמך זה ההתייחסות לכל מי שאינו מכונה בשם היא בלשון זכר. זאת מטעמי נוחות בלבד, והכוונה היא גם לנקבה אם לא מצוין אחרת.

לצורך הכניסה לבית הספר יש להמציא העתק של היתר זה למנהל המוסד

המסגרת שבה נערך המחקר: לימודיה של עורכת המחקר לקראת תואר מגיסטר בטכניון

המנחה של העבודה: ד"ר עינת הד מצויינים

עיקרי המרכיבים של המחקר לעניין היתר זה

הנבדקים:
קבוצה א': תלמידי כיתות ז'-ט' שהשתתפו בשנת הלימודים תשע"ט בשיעורי עזר במתמטיקה בהם נעשה שימוש במשחקים מתמטיים, תוך תיעוד השיעורים באמצעות צילומי ווידיאו

קבוצה ב': תלמידי כיתות ז'-ט' שלא השתתפו בשנת הלימודים תשע"ט בשיעורי העזר הנ"ל

הפעולות המחקריות:

- בקבוצת מחקר ב' תופעל במחצית הראשונה של השנה תכנית התערבות במסגרת קבוצתית פעם בשבוע בתום יום הלימודים, במהלכה יינתנו שיעורי עזר במתמטיקה תוך שימוש במשחקי למידה מתמטיים. שיעורי התכנית יתועדו באמצעות צילומי ווידיאו.
- בקבוצת מחקר א' תיצור עורכת המחקר עותקים של צילומי ווידיאו שנערכו בשנת הלימודים תשע"ט בהשתתפות הנבדקים לצורך ניתוחם המחקרי.
- כלל הנבדקים יתבקשו למלא שאלון עמדות בנוגע להוראה משחקית במתמטיקה.
- כלל הנבדקים יתבקשו להשתתף בראיון בנושא עליו נשאלו במסגרת השאלון. הריאיון יתועד באמצעות הקלטה קולית.
- לפני הפעלת תכנית ההתערבות ובסיומה, יתבקשו הסייעים הנבדקים למלא שאלון הנוגע למידע שרכשו אודות תת"ח וכן לגבי עמדותיהם כלפי הטמעת טכנולוגיה מסייעת לתת"ח בבתי הספר.

כלל המידע ייאסף באופן המזהה את הנבדקים בפני עורכת המחקר לצורך הצלבת הנתונים שייאספו על אודות אותו נבדק באמצעים השונים.

הבקשה להפעלת ההתערבות ולאיסוף המידע הנ"ל נבדקה על ידי לשכת המדען הראשי ונמצאה עומדת בכל התנאים הנוגעים בדבר הקבועים בנוהלי משרד החינוך לפעילות מחקרית במערכת החינוך.[1] לאור

1. הנהלים מפורסמים בחוזר המנהלת הכללית – הוראות קבע – עה/9(ב) מחודש מאי 2015 ומופיעים גם באתר לשכת המדען הראשי.

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זאת הוחלט להתיר את ביצוע כלל הפעולות שלעיל בקרב הנבדקים שלעיל בבתי ספר ממלכתיים במחוז תל אביב.

ההיתר כפוף לאלה:

א. הסכמתו העקרונית של מנהל בית הספר להפעלת ההתערבות המבוקשת להעברה בקבוצת מחקר ב' במוסד שבניהולו.

ב. כל התנאים הנוספים המפורטים להלן, אשר אותם על עורכת המחקר לקיים בשיתוף עם מנהל המוסד החינוכי הנוגע בדבר, אלא אם כן נקבע אחרת בתנאי עצמו:

1. המנהל של המוסד החינוכי שבמסגרתו מתוכנן המחקר להיערך, התלמידים המועמדים להיכלל במחקר וכן הוריהם של התלמידים- כל אלה אינם תלויים בעורכת המחקר ובמנחה של עבודתה מסיבה כלשהי ובכלל זה כפיפות בתפקיד.

2. הפעלת ההתערבות ואיסוף המידע על אודותיה ייערכו אך ורק על ידי עורכת המחקר עצמה.

3. בפעולות המחקריות ייטלו חלק אך ורק אלה מבין התלמידים, שהוריהם נתנו את הסכמתם בכתובים להכללת בנם/בתם בכלל הפעולות המבוקשות. לצורך קיום תנאי זה תיערכנה פעולות אלה:

א. עורכת המחקר תספק להנהלת המוסד החינוכי את הכמות הנחוצה של מכתבי הפנייה להורים בנושא (בנספח 1 הנלווה להיתר זה מוצג נוסח המכתב וכתב ההסכמה, החתומים דיגיטלית על ידי לשכת המדען הראשי, שאותם יש להפיץ בקרב ההורים, שילדם מועמד להכלל בקבוצת מחקר א'. בנספח 2 הנלווה להיתר זה מוצג נוסח המכתב וכתב ההסכמה, החתומים דיגיטלית על ידי לשכת המדען הראשי, שאותם יש להפיץ בקרב ההורים, שילדם מועמד להכלל בקבוצת מחקר ב').

ב. הנהלת המוסד החינוכי תסדיר את הפצת המכתבים להורים.[2]

ג. ההורים המסכימים להכללת ילדם באיסוף המידע המבוקש יחזירו את כתב ההסכמה החתום על ידם להנהלת המוסד החינוכי, וממנה תאסוף עורכת המחקר את הטפסים הנ"ל ותמסור העתק מהם לשמירה במוסד.[3]

4. תלמיד, שהוריו לא הסכימו להכללתו במחקר לא ייפגע בדרך כלשהי בעקבות החלטת הוריו.

5. לקראת איסוף המידע תתאר עורכת המחקר בפני כלל המועמדים להיבדק[4] את הפעולות המתוכננות לביצוע ותבהיר להם את זכותם שלא להשתתף בהן. כמו כן, תבהיר עורכת המחקר לנבדקים את זכותם להפסיק את השתתפותם בכל אחת מהפעולות המחקריות המבוקשות באמצע, מבלי שהם עצמם או ילדם (בהתייחס להורים, שילדם מועמד להיבדק) ייפגעו בעקבות החלטתם באופן כלשהו.[5]

6. כלל זכויותיהם של המועמדים להיבדק ושל הנבדקים עצמם והוריהם ישמרו בקפידה ובכלל זאת זכותם של הנבדקים לפרטיות. במסגרת השמירה על פרטיותם יונחו הנבדקים למסור את השאלון שמילאו ישירות לידי עורכת המחקר.

7. משך הזמן המרבי שיותר לביצוע הפעולות המחקריות המבוקשות ייקבע על ידי מנהל בית הספר. זאת, בהתאמה להבהרות שרשומות בנושא במכתבי הפנייה להורי התלמידים המועמדים להיבדק.

2. בהעדר הסכמת ההורים, אין לאפשר לעורכת המחקר גישות לפרטים מזהים כלשהם על אודות ההורים, כולל לאלה שהיו נחוצים לה לצורך הפצת המכתבים בכוחות עצמה.
3. המסמכים הנ"ל יישמרו לצמיתות על-ידי הנהלת המוסד החינוכי, ובאחריותה, במקום מאובטח.
4. כמועמד להיבדק מוגדר מי, שהוריו נתנו בכתובים את הסכמתם להכללתו במחקר.
5. זכויות אלה כמו גם זכויות הורי התלמידים בקשר להכללת ילדם בפעולות המחקריות מובהרות להורים במסגרת המכתב המופץ בקרבם המצורף להיתר זה.

8. **מילוי השאלון ייערך במסגרת קבוצתית במהלך יום הלימודים, בתאום עם המורה.**

9. **כלל הפעולות המחקריות ייערכו בשפת ההוראה העיקרית הנהוגה במוסד החינוכי ובשפת התקשורת העיקרית של בית הספר עם ההורים, בהתאמה לפעולה.**[6]

10. **בעת הכניסה לבית הספר, על עורכת המחקר לשמור על הופעה ההולמת את המקובל במוסד.**

<u>עוד יובהר כדלהלן:</u>

- עורכת המחקר והמנחה של עבודתה התחייבו בכתובים לפני לשכת המדען הראשי:
  א. לא לפרסם את ממצאי המחקר באופן שיאפשר את זיהוי הנבדקים.
  ב. לשמור בקפדנות על החיסיון של צילומי הווידיאו, הקלטות השמע וכל יתר הנתונים המזוהים שייאספו במסגרת המחקר, להשמיד את צילומי הווידיאו שישמשו לצורך המחקר ולהשמיט לצמיתות את פרטי הזיהוי של הנבדקים מכל יתר הנתונים הנ"ל מיד עם תום עיבוד הנתונים ובכל מקרה לא יאוחר מיום 31.12.2025, במועד המוקדם מבין השניים.
- היתר זה תקף אך ורק בנוגע לגרסתה של ההצעה על כליה שעמידתם בכללי משרד החינוך אושרה על ידי לשכת המדען הראשי, בכפוף לתנאים המפורטים לעיל.
- אם קיימות זכויות יוצרים בנוגע להתערבות ו/או לכלי המחקר או בנוגע לתשובות או לתוצרים האחרים של הנבדקים הנוצרים במסגרת איסוף המידע ו/או הנמסרים לעורכת המחקר, באחריותן הבלעדית של עורכת המחקר והמנחה של עבודתה להסדיר את הנושא בהתאם לכל הוראות החוק/ים הנוגע/ים בדבר.
- אין במסמך זה משום חיווי דעה של לשכת המדען הראשי על איכותו של המחקר.
- לא נדרש היתר נפרד מטעם המחוז.

ד"ר איתי אשר
המדען הראשי

תפוצה:
גבי פאולה לוי, הטכניון
ד"ר עינת חד מצויינים, הטכניון
מנהלת מחוז תל אביב

6. במקרה ששפת ההוראה ו/או שפת התקשורת של בית הספר עם הורי התלמידים אינה עברית, האחריות לתרגום כלי המחקר והמכתבים להורים והתאמתם לנוסח שהותר בשפה העברית חלה על עורכת המחקר ועל המנחה של עבודתה.

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1. תקציר

מערכות החינוך העולמיות תומכות ומעודדות את המעבר מהוראה מסורתית להוראה שבה התלמיד במרכז . הוראה זו מקדמת את עצמאות הלומד, מעורבות הלומד, למידת חקר ופיתוח מיומנויות הלומד במאה ה- 21 כמו חשיבה ביקורתית ועבודה בקבוצות. שיטות הוראה רבות הנחשבות לכאלה שהן ממוקדות-לומד, כגון, הוראה מבוססת פרויקט, הוראה מבוססת שיח, הוראה מבוססת על פתרון בעיות וכיוצא באלה תפסו תאוצה, ודלקו בעקבותיהם מחקרים שבדקו את יתרונות ההוראה שבשיטות אלו, את אופן ההוראה הנדרשת מהמורה וכן את תהליך הלמידה של הלומד. הרבה פחות נחקר בנוגע להוראה המבוססת על משחקים.

מחקר זה מתמקד בשימוש במשחקים מתמטיים חינוכיים שבהם השחקנים משחקים פנים אל פנים בתוך כיתת הלימוד, במסגרת לימודי היום השגרתיים בחטיבת הביניים. נושא זה נחקר לעיתים נדירות בספרות המקצועית, ולרוב מתבסס על שיטות מחקר כמותיות.

מחקרים קודמים אודות משחקים דיגיטאליים מצאו כי הישגי התלמידים, המוטיבציה שלהם, והגישה כלפי מקצוע המתמטיקה משתפרים באמצעות למידה שמבוססת על משחקים. מרבית המחקרים הללו התבססו על שיטות מחקר כמותיות שנעשו באמצעות קבוצת ניסוי וקבוצת ביקורת והמשתנים נמדדו באמצעות שאלונים (לפני ואחרי הניסוי) ובאמצעות מבחני הישגים (לפני ואחרי הניסוי).

המחקר הנוכחי מציע כלי מתודולוגי חדש לניתוח למידה באמצעות משחק, המתמקד בתהליכי ההשתתפות של הלומד. כלי זה בוחן את מאפייני ההשתתפות החקירתית של התלמידים במהלך למידה באמצעות משחק. הכלי מתבסס על התיאוריה הקומוגניטיבית ששורשיה נטועים בגישות הסוציו-תרבותיות, ואשר רואה בחשיבה ובתקשורת את הבסיס ללמידה. למידה מוגדרת כשינוי בשיח של הלומד. במהלך הלמידה, ההשתתפות של הלומד נעה מהשתתפות ריטואלית בעיקרה אל עבר השתתפות חקירתית יותר, בהנתן הזדמנויות למידה מתאימות. השתתפות ריטואלית מתמקדת בביצוע נוקשה של הליכים מוכרים מראש, או בחיקוי של אחרים המבצעים את אותם הליכים, ואילו השתתפות חקירתית מתמקדת בפיתוח נרטיבים של חדשים ללומד והיא מתאפיינת בגמישות ולקיחת אחריות על תהליך הלמידה. מחקר זה מתמקד בעיקר במאפיינים הבאים של השתתפות חקירתית שאותם התאמתי להקשר המשחקי: סוכנות (אופן המעורבות והעצמאות של תלמידים תוך כדי בחירתם ברוטינות לשם עשייה מתמטית), יישומיות (השימוש בנרטיבים ורוטינות מוכרות על מנת לפתור בעיות מתמטיות), קישוריות (האופן שבו התוצאה של צעד בפתרון מובילה אל הצעד הבא בפתרון), הנמקה (האופן שבו תלמידים מצדיקים את הרוטינות והנרטיבים המתמטיים) וגמישות (פתרון בעיה מתמטית באמצעות מספר פרוצדורות שונות זו מזו).

בנוסף, מחקר זה בודק אלו מאפיינים של המשחק עצמו, הקשורים לעיצוב המשחק ולחוקיו, מקדמים את ההשתתפות החקירתית של הלומד ואלו מעכבים אותה.

המחקר עשה שימוש בצילומי וידאו ותמלולים של חמש קבוצות תלמידים בכיתה ט' אשר שיחקו בשלושה משחקים מתמטיים שונים במהלך שיעורי המתמטיקה בבית הספר. מדובר ב 14 משתתפים שחלקם לומדים בכיתה מתקדמת (הקבצה א') ושני תלמידים מתקשים הלומדים בכיתת תל"מ. אחד משני תלמידים אלה משולב בכיתת תל"מ מהחינוך המיוחד. הצילומים תומללו במדויק, כולל תנועות ידיים ואינטונציית דיבור. צילומי הוידאו והתמלולים נותחו במטרה לבחון את מאפייני ההשתתפות החקירתית של התלמידים במהלך המשחק, וכן את הפוטנציאל של המשחק לזמן לשחקנים הזדמנות להשתתפות חקירתית.

הממצאים מלמדים כי התלמידים השתתפו באופן חקירתי בכל שלושת המשחקים, אך מתוך חמישה מאפייני השתתפות בלטו שלושה: סוכנות, קישוריות ויישומיות. נמצאו דרכי השתתפות שונות עבור כל מאפיין השתתפות חקירתי (במחקר זה לכל דרך השתתפות שונה קראנו "סוג"). מאפיין הסוכנות בלט בכל המשחקים והקבוצות. נמצאו ארבעה סוגים של סוכנות (תכנון, הבהרות, יצירה וביצוע של

רוטינות מתמטיות). נמצאו שני סוגים מעניינים של יישומיות שכללו למידה ויישום של רוטינות חדשות במהלך המשחק. אחד מהם הינו למידה מתלמיד אחר בעוד השני כולל למידה מהעיצוב המשחקי עצמו. הקישוריות נמצאה הן במבנה גלובלי של קישוריות והן בתתי-רוטינות שמרכיבות כל שלב ברוטינה הגלובלית. נמצא כי העיצוב של כל אחד משלושת המשחקים והחוקים שלהם, עודדו את הקידום של כל אחד משלושת מאפייני השתתפות אלו.

עם זאת, למרות שנמצא כי ההנמקה אינה נדרשת בחוקי המשחק ולא מקודמת על ידי עיצוב המשחק, ניתן היה להבחין כי תלמידים מצדיקים את ההליכים המתמטיים שלהם על פי הצורך המשחקי והמתמטי שעלה מצד השחקנים במהלך המשחק. בנוסף, נמצא שמאפיין ההשתתפות של גמישות אינו נדרש בחוקי המשחק וגם אינו נמצא בפועל בקרב המשתתפים במהלך המשחק.

ממצאים בנוגע לעיצוב המשחק וחוקיו מלמד כי ישנם מרכיבים עיצוביים ומשחקיים המקדמים השתתפות חקירתית של סוכנות (על ידי מספר השחקנים, וקלפים משותפים לדוגמה), יישומיות (על ידי עיצוב משחקים המכילים תוכן מתמטי מוכר למתלמידים) וקישוריות (מהלכים בהם המשחק דורש יותר מצעד אחד על מנת להתקדם במשחק). המרכיבים העיצוביים וכללי המשחקים במחקר אינם מקדמים הנמקה ואף מעכבים השתתפות גמישה (לדוגמה על ידי משוב ממוחשב).

התרומה העיקרית של המחקר היא בפירוט וההבנה של אופני ההשתתפות של תלמידים במהלך משחקים מתמטיים המאפשרת להבין איך נראות הזדמנויות למידה הלכה למעשה בזמן המשחק. למחקר תרומות יישומיות והמלצות בנוגע לשימוש במשחקים מתמטיים עבור מורים ועבור אנשי חינוך המעוניינים בפיתוח ועיצוב משחקים מתמטיים. ההמלצות נובעות מהממצאים וכוללות דרכים להעשיר ולשפר את ההשתתפות החקירתית של התלמידים ואופן היישום של המשחק במהלך השיעור. כמו כן, למחקר תרומה מתודולוגית בכך שהוא מציע לי מתודולוגי, המבוסס על הגישה הקומוגניטיבית, לחקירת אופן ההשתתפות של תלמידים בעת משחק מתמטי.

המחקר נעשה בהנחיית פרופ"ח עינת הד-מצויינים ופרופ"ח טלי נחליאלי בפקולטה לחינוך מדעי טכנולוגי.

תזה זו מוקדשת באהבה אין קץ ל**מתאו** שליווה אותי בכל השעות הקטנות של הלילה.

ברצוני להביע את תודתי העמוקה למשפחתי וחברתי האהובים על הסבלנות, העידוד, החשיבה החיובית וההכלה לאורך כל התקופה.

מאפיינים של השתתפות חקירתית של תלמידים במהלך משחקים מתמטיים במהלך שיעורי המתמטיקה בחטיבת הביניים

חיבור על מחקר לשם מילוי חלקי של הדרישות לקבלת התואר מגיסטר למדעים בהוראת הטכנולוגיה והמדעים

פאולה לוי

הוגש לסנאט הטכניון – מכון טכנולוגי לישראל

אלול תשפ"א חיפה אוגוסט 2021