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Mapping Objectification in Early Algebraic Discourse

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Research Report

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MOTIVATION & BACKGROUND

- The beginning of algebra learning poses multiple challenges to students (Kieran, 2022)
  - E.g. The challenge of solving equations such as ax+b=cx+d (Radford, 2014)
- However, there is absence of tools for mapping what students are doing in the face of these challenges.
- Commognition - useful for mapping students' difficulties, as it provides a non-deficit lens on learning (Ben-Yehuda et al., 2005; Heyd-Metzuyanim, 2016)

THEORETICAL LENS – COMMOGNITION

WHAT IS LEARNING?

Learning is the change in one's discourse

- a change in the way one communicates.

Discourses are made up of:

- Keywords and signifiers (2, f(x), m)
- Routines for endorsing narratives about these objects
  - Adding, subtracting, balancing an equation
- Meta-rules governing these routines, when to apply them, and which routines are appropriate
  - Often implicit and only enacted

Sfard, A. (2008). *Thinking as communicating*. Cambridge University Press.

COMMOGNITIVE CONCEPTUALIZATION OF OBJECTIFICATION

- Objects are the central component of a discourse
  - Arithmetic discourse – discourse about numbers
  - Early algebraic discourse – discourse about indeterminates (unknowns, variables)
  - Algebraic discourse – discourse about functions
- So far, objectification has been studied in the context of:
  - Numbers (Sfard & Lavie, 2005; Lavie & Sfard, 2019)
  - Functions (Caspi & Sfard, 2012, Nachlieli & Tabach, 2012; Sfard & Linchevsky, 1994)
  - Infinite quantities (Kim et al. 2012)

WHAT ARE SOME OF THE EXPECTED METARULES IN EARLY ALGEBRAIC DISCOURSE?

- The participants should be answering new questions:
  - Given the specific result of a calculation what was the number acted upon?
    - $x + 3 = 8$
    - $7x = 14$
  - Given a relation between two (or more) numbers, what is the unknown?
    - $x + 3 = 2x - 6$

Caspi & Sfard (2012); Filloy & Rojano (1989); Radford (2014)

RESEARCH QUESTION

How do we map objectification of non-specific numbers in students' early algebraic discourse?

THEORETICAL DEVELOPMENT: WHAT DOES IT MEAN TO OBJECTIFY UNKNOWNS?

An unknown **number**

- Adopts new signifiers (x, m, n …), instead of "two", "third", "million"
  - But can also be verbal ("the number")

Objectifying unknowns means:

- Learning to attribute all the familiar characteristics of numbers to the new signifiers
- Adopting new meta-rules about the questions that can be asked on the unknown

METHODOLOGY – DATA

- Early Algebraic Discourse Profile (EADP) interviews.
- 10 students, 7th grade, various achievement levels.
- Think-aloud format, individual interviews
- Focus on 'I thought of a number' problem.

I thought of a specific number. If I multiply it by 7 and subtract from the product 54, I will get the number I was thinking of. What is the number I was thinking of? Explain how you solved it.

ANALYTICAL QUESTIONS AROUND ATTRIBUTING TO A SIGNIFIER THE PROPERTIES OF NUMBERS

1. Is there a signifier (symbolic or non-symbolic) in the discourse for a non-specific number?
2. Is the signifier (symbolic or non-symbolic) something that can be acted upon?
3. Can the result of an action on a non-specific number be a non-specific number?

DETECTING META-RULES IN STUDENTS' DISCOURSE AROUND NON-SPECIFIC NUMBERS

- Focused on students' questions and exclamations as they halted or got stuck
- Asked:
  - What are the declared and implied constraints on the routines appropriate for the task?
  - What are the student's expectations regarding the routines and when are they disrupted?

FIRST CASE - ERAN

I thought of a specific number. If I multiply it by 7 and subtract from the product 54, I will get the number I was thinking of. What is the number I was thinking of? Explain how you solved it.

No.	Spkr	What is said ((what is done))
182	Eran	(Reads the question), like, like A times 7 minus 54? Like that's the exercise?
183	Interv.	OK
184	Eran	I don't know which number you're thinking of
185	Interv	That's what you need to…
186	Eran	You could think of any number
	…	
190	Eran	Where would I know what number you thought of? There's not enough data for that
191	Inter.	What's that A you wrote there?
192	Eran	Unknown. We don't know what it is
	…	
194	Eran	That's the number she's thinking of. The number she's thinking of is A.
.	….	
198	Eran	I can't solve it because I could do that A equals a million and then it's a million times 7 (…) It's impossible.

Metarule: unknowns function as "place-holders" for specific numbers. The only way to find them is if you reveal what they are.

There's a signifier for a non-specific number

SECOND CASE - LIAT

I thought of a specific number. If I multiply it by 7 and subtract from the product 54, I will get the number I was thinking of. What is the number I was thinking of? Explain how you solved it.

No.	Speaker	What is said ((what is done))
285	Liat	(reads) I thought of a specific number that if I multiply it by subtract.. (writes) x, 7 x, 7 x minus 54
…		
287	Liat	What did I do? But what is the x … uals x perhaps?
288	Int	Read the question, you seem a bi…
289	Liat	I don't know, I don't, I don't ((era… …, if I multiply it by seven,
290	Liat	x times 7 and I subtract… x… x is the specific number.
291	Liat	If I multiply it by 7 and subtract 54 from the product,
292	Liat	But what do we do if I subtract (from) the …oduct 54? How do we write it as an exercise?

Metarule: uknowns function as "place-holders" for specific numbers. They can be found through calculations ("exercises"), similarly to specific numbers

There's a signifier for a non-specific number

X · 7

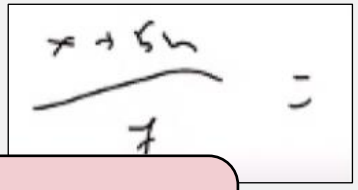
X · 7 − 54

X Can be acted upon

THIRD CASE - GIL

I thought of a specific number. If I multiply it by 7 and subtract from the product 54, I will get the number I was thinking of. What is the number I was thinking of? Explain how you solved it.

No.	Speaker	What is said ((what is done))
192		Gil reads the question, asks a clarifying question, the interviewer answers.
193	Gil	Okay, alright. So, I'll do it ((writes and talks at the same time))
		x plus fifty-four divided by seven
194	Gil	Wait[illegible]hat? ((erases what he wrote))
		he found the ans
212-218	Gil	I tried to put a number that was like a
		that it didn't make sense, a[illegible]
		and then I got that it didn't make sens
		the (number with) biggest single digits



X Can be acted upon

There's a signifier for a non-specific number

Metarule: uknowns function as "place-holders" for specific numbers. They can be found through "excercise" or through various manipulations, constrained by the numbers they relate to.

FOURTH CASE - TOM

I thought of a specific number. If I multiply it by 7 and subtract from the product 54, I will get the number I was thinking of. What is the number I was thinking of? Explain how you solved it.

No.

and writes an equation

32 ...u wrote?

33

62

63 Tom No, I just don't understa...

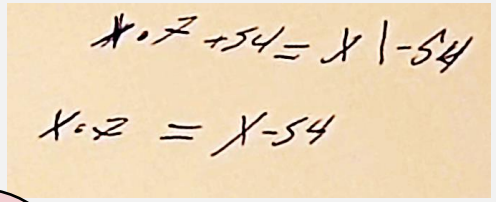
54, 54, can be equal to t...

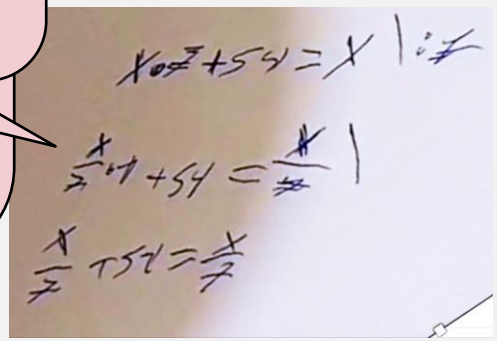
can be a non-specific number

Metarule: uknowns can be found through equations

Metarule: However, some things can be done on numbers (e.g. subtract from both sides) that cannot be done on X (subtract x from both sides)

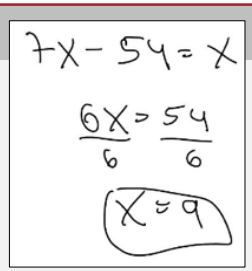
However: 7x divided by 7 becomes x/7;
X behaves differently than a number





FIFTH CASE - ALON

I thought of a specific number. If I multiply it by 7 and subtract from the product 54, I will get the number I was thinking of. What is the number I was thinking of? Explain how you solved it.



No.	Speaker	What is said ((what is done))
3	Alo	lain)
5-10	Alon	Yes…mmm…I gave the number an unknown which is x, (I) multiplied it (x) by 7, 7 x, and then I subtract from the product 54 and it says we get the number itself. So, I made it (the number) x
11-13	Alon	now I did… 7 x…to move it (the x on the right side) to here (to the left side) so that's minus x and so it's 6 x, and then it is moved to here (54 from left to right), so, it's 54.
14		
15		and denominator) by 6
16		

Metarule: uknowns can be found through equations that specify their exact relations

There's a signifier for a non-specific number;
The signifier can be acted upon;
The result of the actions can be a non-specific number

However: numbers and signifiers behave differently in equations than in familiar arithmetic discourse ("moved" and "change signs")

SUMMARY

	Use of symbols	Attributes of Signifiers	Meta-rules about non-specific numbers	Correct solution
Eran	A	Stand as a "place-holder" of a number	Need to be "told". No other way to find them.	-
Liat	7x-54=	Stand as a place-holder; Are acted upon	Can be found through "exercise" (calculation)	-
Gil	(x+54)/7	Stand as a place-holder; Are acted upon	Can be found through relations	-
Tom	7x-54=x	Stand as a place-holder; Are acted upon; Function also as result	Can be found through equations; but can be manipulated "non-numerically"	-
Alon	7x-54=x	Stand as a place-holder; Are acted upon; Function also as result	Can be found through equations; but carry some non-numeric attributes	+

DISCUSSION

- Signifying non-specific numbers and the adoption of new metarules around them go hand in hand
- However, there are places where the meta-rules develop independently of the signifier
  - e.g.: Alon's treatment of the X signifiers agrees with all the properties of numbers
    - However, his meta-rules of "moving sides" in equations do not apply to numbers
  - Tom's meta-rules of finding unknowns with equations are relatively sophisticated
    - However, his use of the signifier X in an equation does not always agree with attributes of numbers.

THEORETICAL CONTRIBUTION

An unknown **number**

- Adopts new signifiers (x, m, n …)
- Should keep all the properties of numbers from the arithmetic discourse

Objectifying unknowns means:

- Learning to attribute all the familiar characteristics of numbers to unknowns
- Saming between the new signifiers and numbers
- Adopting new meta-rules about the questions that can be asked on the unknown

METHODOLOGICAL & EMPIRICAL CONTRIBUTION

- Developed a method for mapping early algebraic discourse according to analytical questions that pertain to:
  - Use of signifiers for non-specific numbers
  - Declared and Implied meta-rules
- We found variation in the level of objectification of non-specific numbers, in 7th graders' discourse

THANK YOU FOR YOUR ATTENTION

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